# Stanford CS224W: Deep Generative Models for Graphs

CS224W: Machine Learning with Graphs
Jure Leskovec, Stanford University
http://cs224w.stanford.edu

# Announcements

- **Homework 2** due today at midnight
- **Homework 3 will be released today by 9PM on our course website**
  - Due Thursday, 11/16 (2 weeks from now)
  - TAs will hold a recitation session for HW 3:
    - Time: TBA
    - Location: Zoom, link will be posted on Ed
    - Session will be recorded

# Announcements

- **Colab 1** grades released
  - Grade distribution and stats announced on Ed
  - Regrade requests open until 11:59 PM Tuesday 11/07

# Motivation for Graph Generation

- So far, we have been **learning from graphs**
  - We assume the graphs are given

Image credit: Medium

**Social Networks**

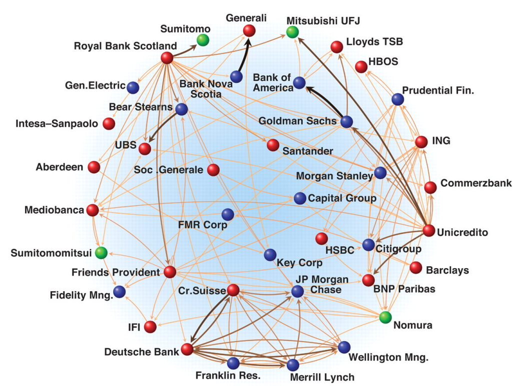



Image credit: Science

**Economic Networks**

Image credit: Lumen Learning

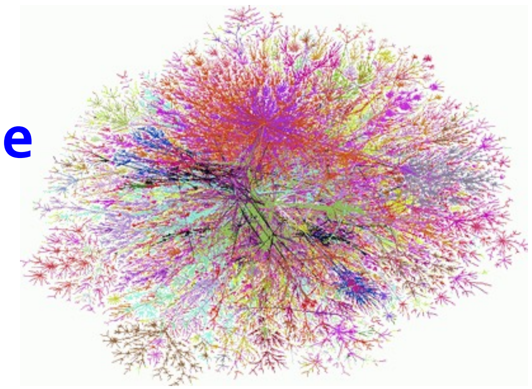
**Communication Networks**

- **But how are these graphs generated?**

# The Problem: Graph Generation

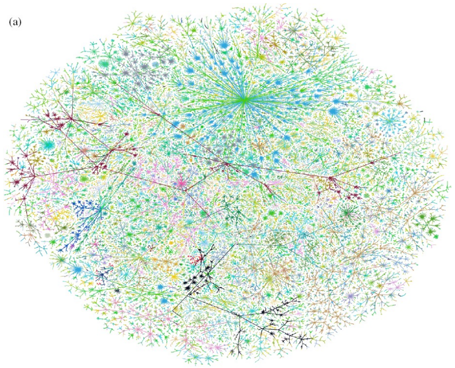
- We want to generate realistic graphs, using **graph generative models**

**Graph Generative Model** → **Generate**



Synthetic graph

**which is similar to** $\approx$



Real graph

- **Applications:**
  - **Drug discovery, material design**
  - **Social network modeling**

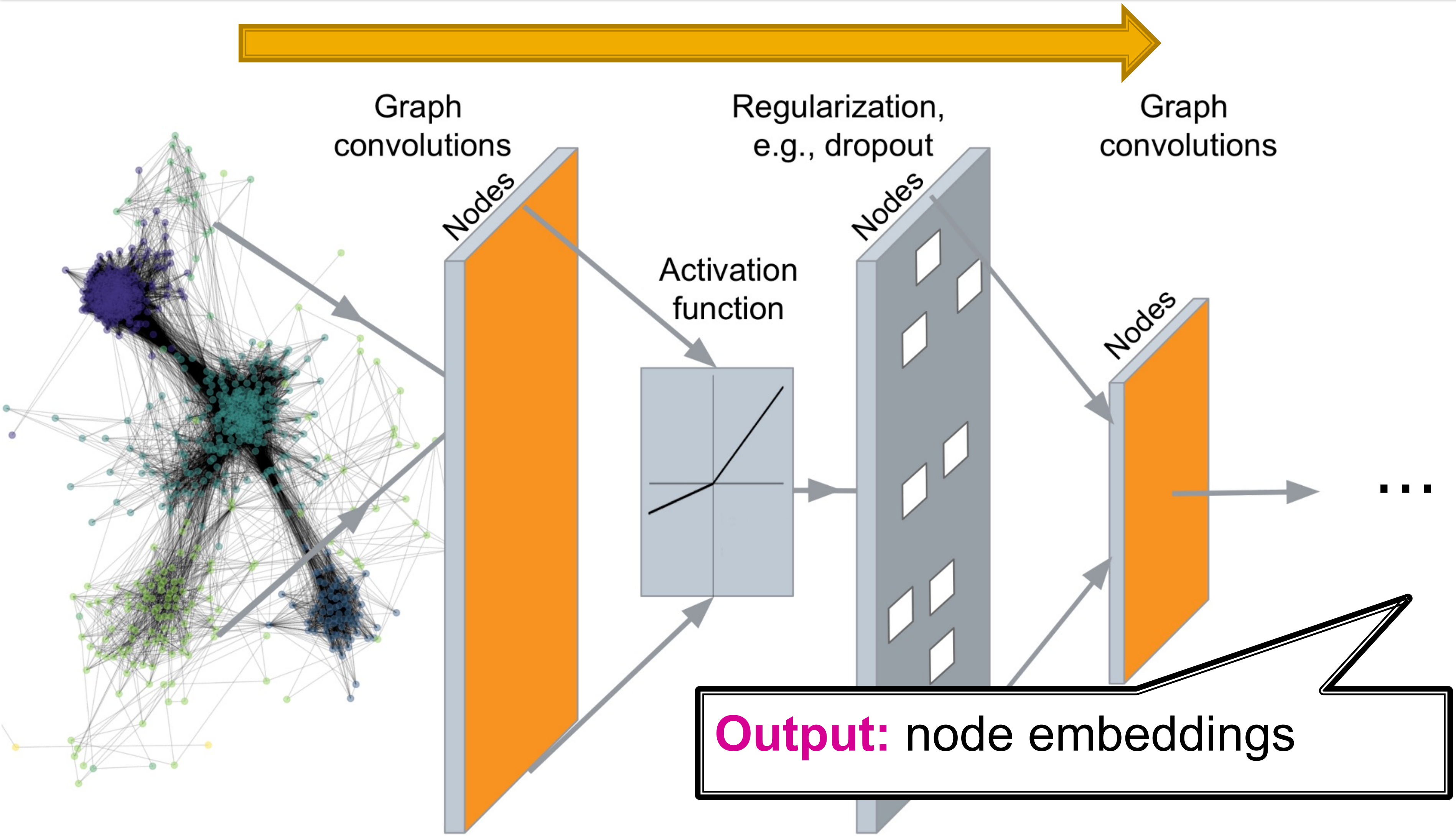
# Why Do We Study Graph Generation

- **Insights** – We can understand the formulation of graphs
- **Predictions** – We can predict how will the graph further evolve
- **Simulations** – We can use the same process to general novel graph instances
- **Anomaly detection** - We can decide if a graph is normal / abnormal

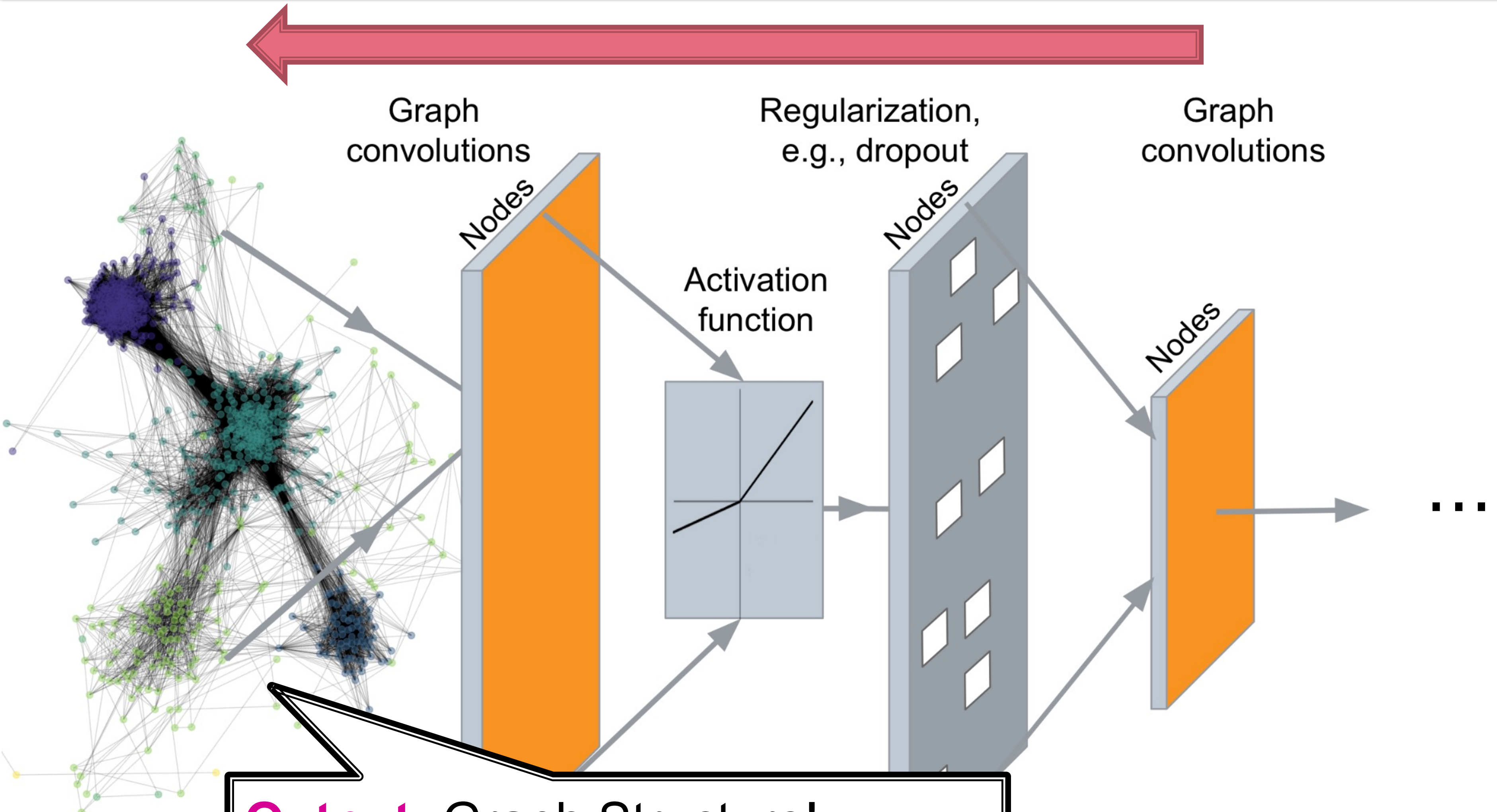
# History of Graph Generation

- **Step 1: Properties of real-world graphs**
  - A successful graph generative model should fit these properties
- **Step 2: Traditional graph generative models**
  - Each come with different assumptions on the graph formulation process
- **Step 3: Deep graph generative models**
  - Learn the graph formation process from the data
  - **This lecture!**

# So far: Deep Graph Encoders



# Today: Deep Graph Decoders




**Output:** Graph Structure!

# Stanford CS224W: Machine Learning for Graph Generation

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# Graph Generation Tasks

## Task 1: Realistic graph generation

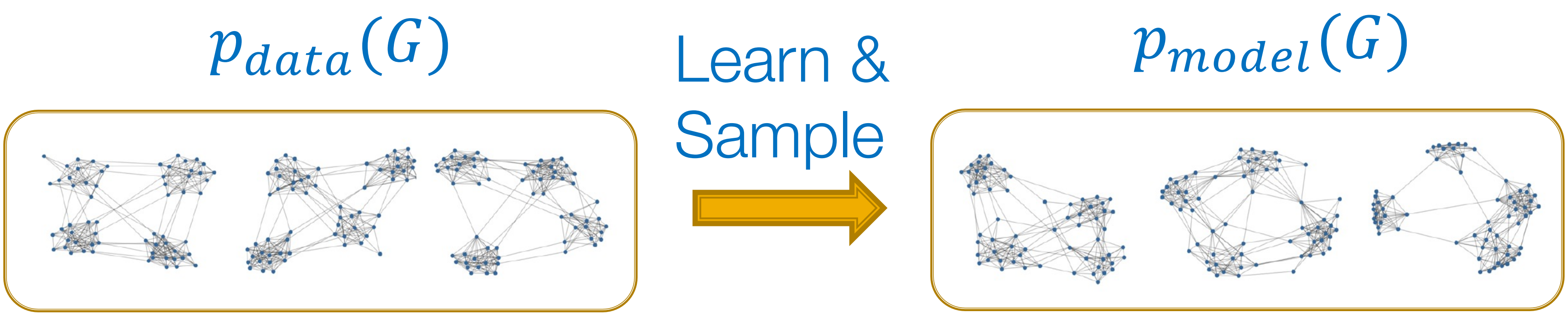
- Generate graphs that are **similar to a given set of graphs** [Focus of this lecture]

## Task 2: Goal-directed graph generation

- Generate graphs that **optimize given objectives/constraints**
  - E.g., Drug molecule generation/optimization

# Graph Generative Models

- **Given:** Graphs sampled from $p_{data}(G)$
- **Goal:**
  - Learn the distribution $p_{model}(G)$
  - Sample from $p_{model}(G)$



# Generative Models Basics

**Setup:**

- Assume we want to learn a generative model from a set of data points (i.e., graphs) $\{\boldsymbol{x}_i\}$
  - $p_{data}(\boldsymbol{x})$ is the data distribution, which is never known to us, but we have sampled $\boldsymbol{x}_i \sim p_{data}(\boldsymbol{x})$
  - $p_{model}(\boldsymbol{x};\theta)$ is the model, parametrized by $\theta$, that we use to approximate $p_{data}(\boldsymbol{x})$
- **Goal:**
  - **(1)** Make $p_{model}(\boldsymbol{x};\theta)$ close to $p_{data}(\boldsymbol{x})$ **(Density estimation)**
  - **(2)** Make sure we can sample from $p_{model}(\boldsymbol{x};\theta)$ **(Sampling)**
    - We need to generate examples (graphs) from $p_{model}(\boldsymbol{x};\theta)$

# Generative Models Basics

## (1) Make $p_{model}(x;\theta)$ close to $p_{data}(x)$

- **Key Principle**: **Maximum Likelihood**
- Fundamental approach to modeling distributions

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\boldsymbol{x} \mid \boldsymbol{\theta})$$

  - Find parameters $\theta^*$, such that for observed data points $\boldsymbol{x}_i \sim p_{data}$ the $\sum_i \log p_{model}(\boldsymbol{x}_i;\theta^*)$ has the highest value, among all possible choices of $\theta$
    - That is, find the model that is most likely to have generated the observed data $x$

# Generative Models Basics

## (2) Sample from $p_{model}(x; \boldsymbol{\theta})$

- **Goal:** Sample from a complex distribution
- The most common approach:
  - **(1)** Sample from a simple noise distribution

$$\boldsymbol{z}_i \sim N(0,1)$$

  - **(2)** Transform the noise $z_i$ via $f(\cdot)$

$$\boldsymbol{x}_i = f(\boldsymbol{z}_i; \theta)$$

  Then $\boldsymbol{x}_i$ follows a complex distribution
- **Q: How to design $f(\cdot)$?**
- **A**: Use Deep Neural Networks, and train it using the data we have!

# Deep Generative Models

## Auto-regressive models:

- $p_{model}(\boldsymbol{x};\boldsymbol{\theta})$ is used for **both density estimation and sampling** (remember our two goals)
    - Other models like Variational Auto Encoders (VAEs), Generative Adversarial Nets (GANs) have 2 or more models, each playing one of the roles
- **Idea: Chain rule.** Joint distribution is a product of conditional distributions:

$$p_{model}(\boldsymbol{x};\theta) = \prod_{t=1}^{n} p_{model}(x_t | x_1, \dots, x_{t-1};\theta)$$

- E.g., $\boldsymbol{x}$ is a vector, $x_t$ is the $t$-th dimension; $\boldsymbol{x}$ is a sentence, $x_t$ is the $t$-th word.
- **In our case:** $x_t$ will be the $t$-th action (add node, add edge)

# Stanford CS224W: GraphRNN: Generating Realistic Graphs

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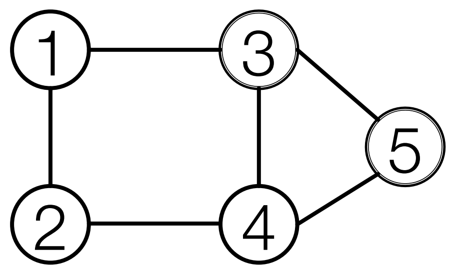
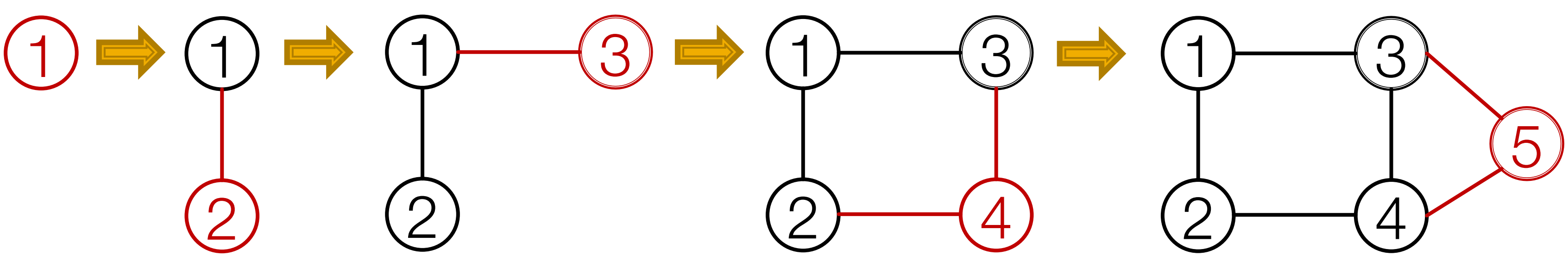
# GraphRNN Idea

## Generating graphs via sequentially adding nodes and edges

Graph $G$




Generation process $S^{\pi}$

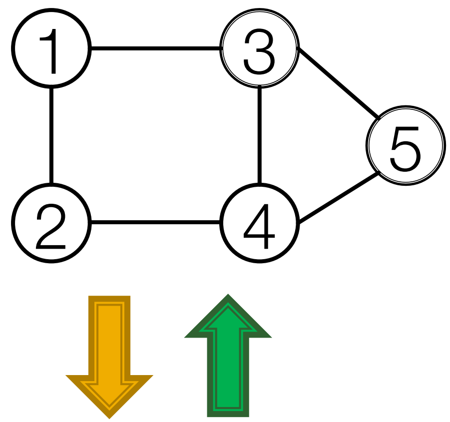



GraphRNN: Generating Realistic Graphs with Deep Auto-regressive Models. J. You, R. Ying, X. Ren, W. L. Hamilton, J. Leskovec. *International Conference on Machine Learning (ICML)*, 2018.

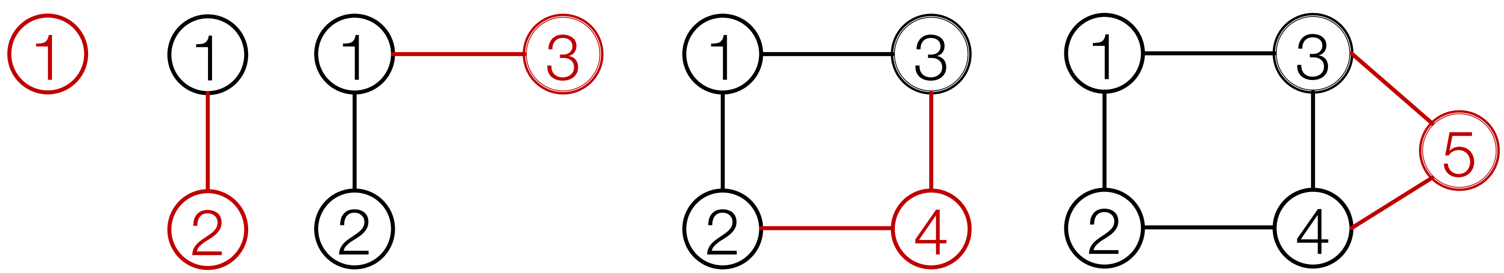
# Model Graphs as Sequences

Graph $G$ with node ordering π can be uniquely mapped into a sequence of node and edge additions $S^{\pi}$

Graph $G$ with node ordering $\pi$:




Sequence $S^{\pi}$:



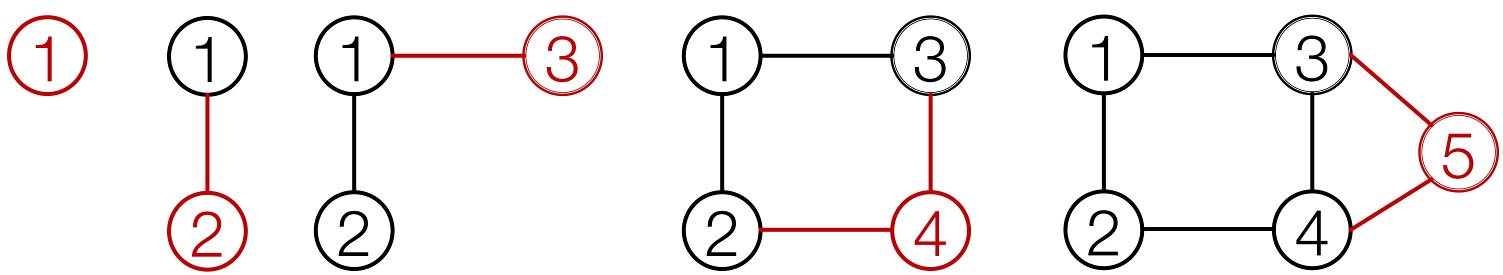

$$S^{\pi} = ( S_1^{\pi} , S_2^{\pi} , S_3^{\pi} , S_4^{\pi} , S_5^{\pi} )$$

# Model Graphs as Sequences

The sequence $S^{\pi}$ has **two levels**
(**$S$ is a sequence of sequences**):

- **Node-level:** add nodes, one at a time
- **Edge-level:** add edges between existing nodes

- **Node-level:** At each step, a new node is added



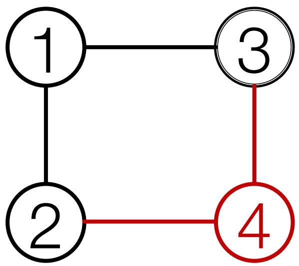

$$S^{\pi} = ( S_1^{\pi}, S_2^{\pi}, S_3^{\pi}, S_4^{\pi}, S_5^{\pi} )$$

"Add node 1" ... "Add node 5"

# Model Graphs as Sequences

The sequence $S^{\pi}$ has **two levels:**

- Each Node-level step is an edge-level sequence
- **Edge-level**: At each step, add a new edge




$S_4^{\pi}$

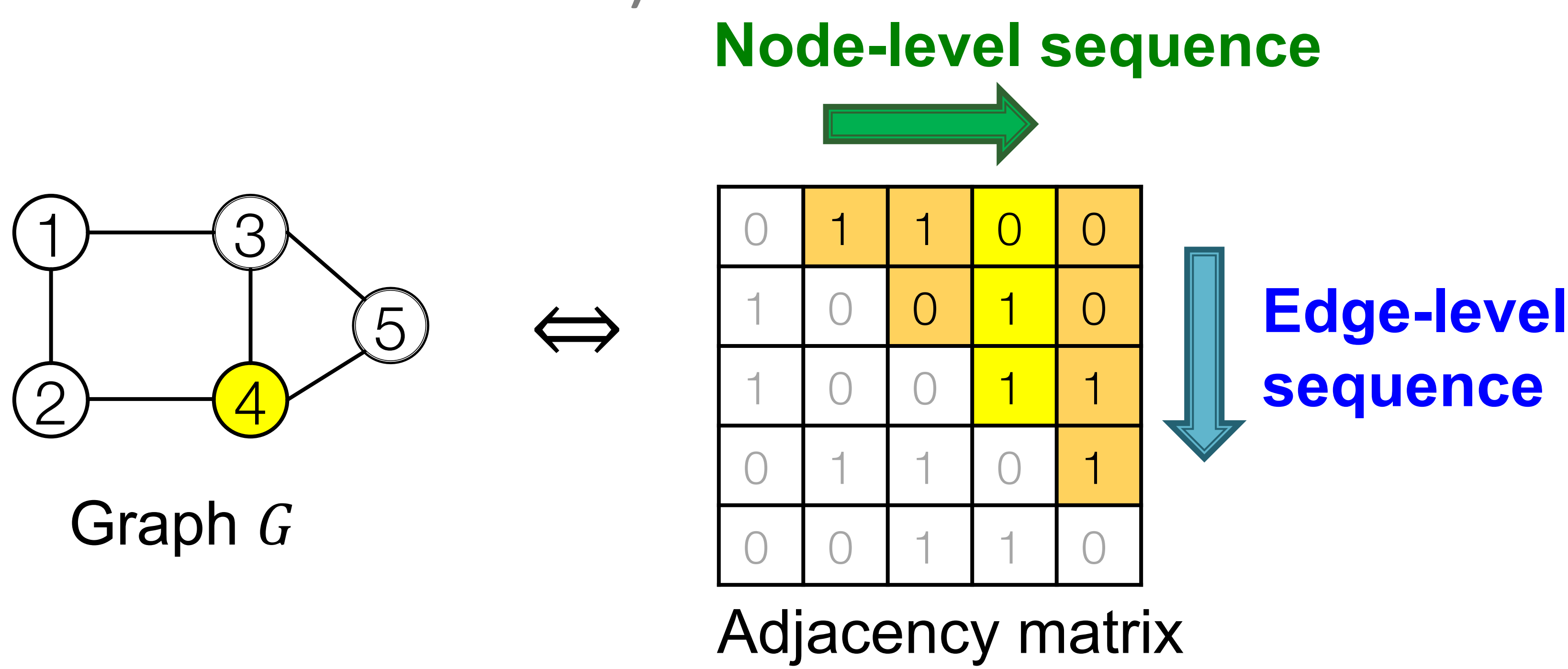
$$S_4^{\pi} = ( \quad S_{4,1}^{\pi} \quad , \quad S_{4,2}^{\pi} \quad , \quad S_{4,3}^{\pi} \quad )$$

"Not connect 4, 1" "Connect 4, 2" "Connect 4, 3"

0 1 1

# Model Graphs as Sequences

- **Summary: A graph + a node ordering = A sequence of sequences**
- Node ordering is randomly selected (we will come back to this)

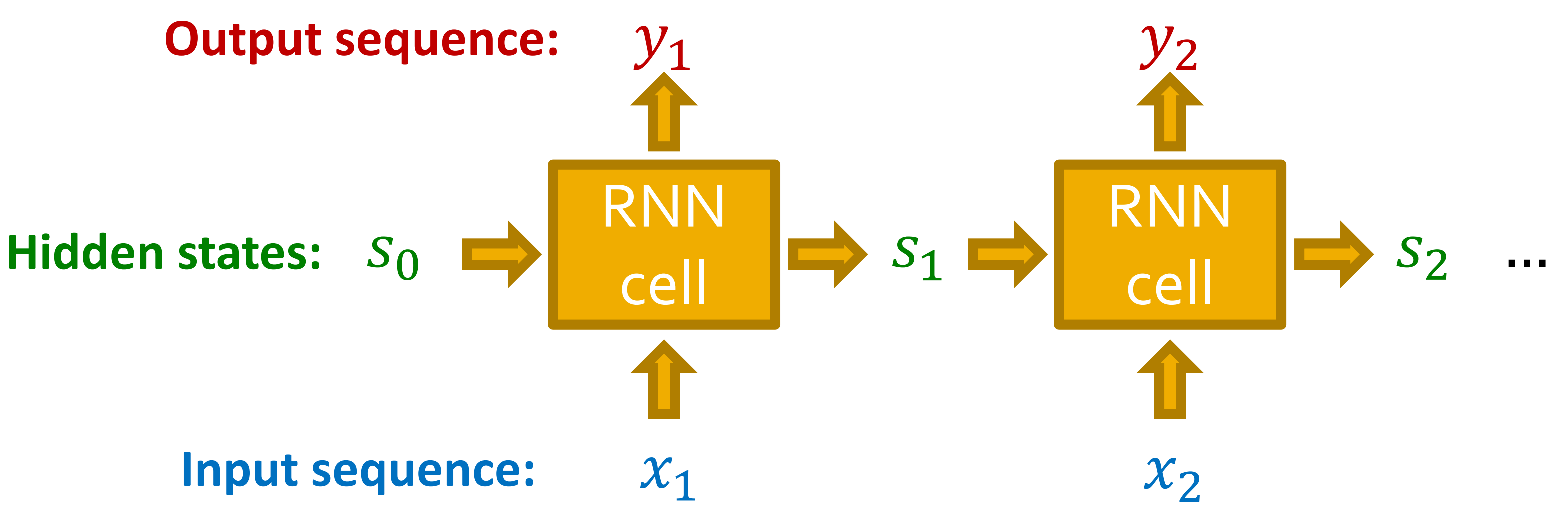


# Model Graphs as Sequences

- **We have transformed graph generation problem into a sequence generation problem**
- **Need to model two processes:**
  - **1)** Generate a state for a new node (Node-level sequence)
  - **2)** Generate edges for the new node based on its state (Edge-level sequence)
- **Approach**: Use **Recurrent Neural Networks (RNNs)** to model these processes!

# Background: Recurrent NNs

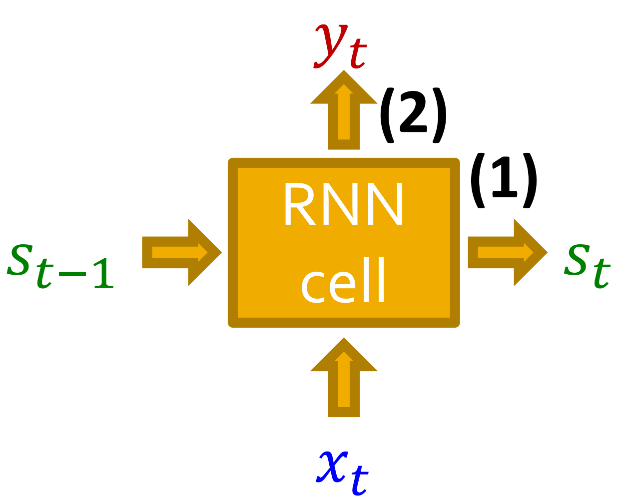
- RNNs are designed for **sequential data**
  - RNN sequentially takes **input sequence** to **update its hidden states**
  - The **hidden states** summarize all the information input to RNN
  - The update is conducted via **RNN cells**



# Background: Recurrent NNs

- $s_t$: **State** of RNN after step $t$
- $x_t$: **Input** to RNN at step $t$
- $y_t$: **Output** of RNN at step $t$
- **RNN cell:** $W, U, V$: Trainable parameters

In our case $s_t$, $x_t$ and $y_t$ will be scalars (edge probabilities)




**The RNN cell:**

**(1) Update hidden state:**

$$s_t = \sigma(W \cdot x_t + U \cdot s_{t-1})$$

**(2) Output prediction:**

$$y_t = V \cdot s_t$$

- **More expressive cells:** GRU, LSTM, etc.

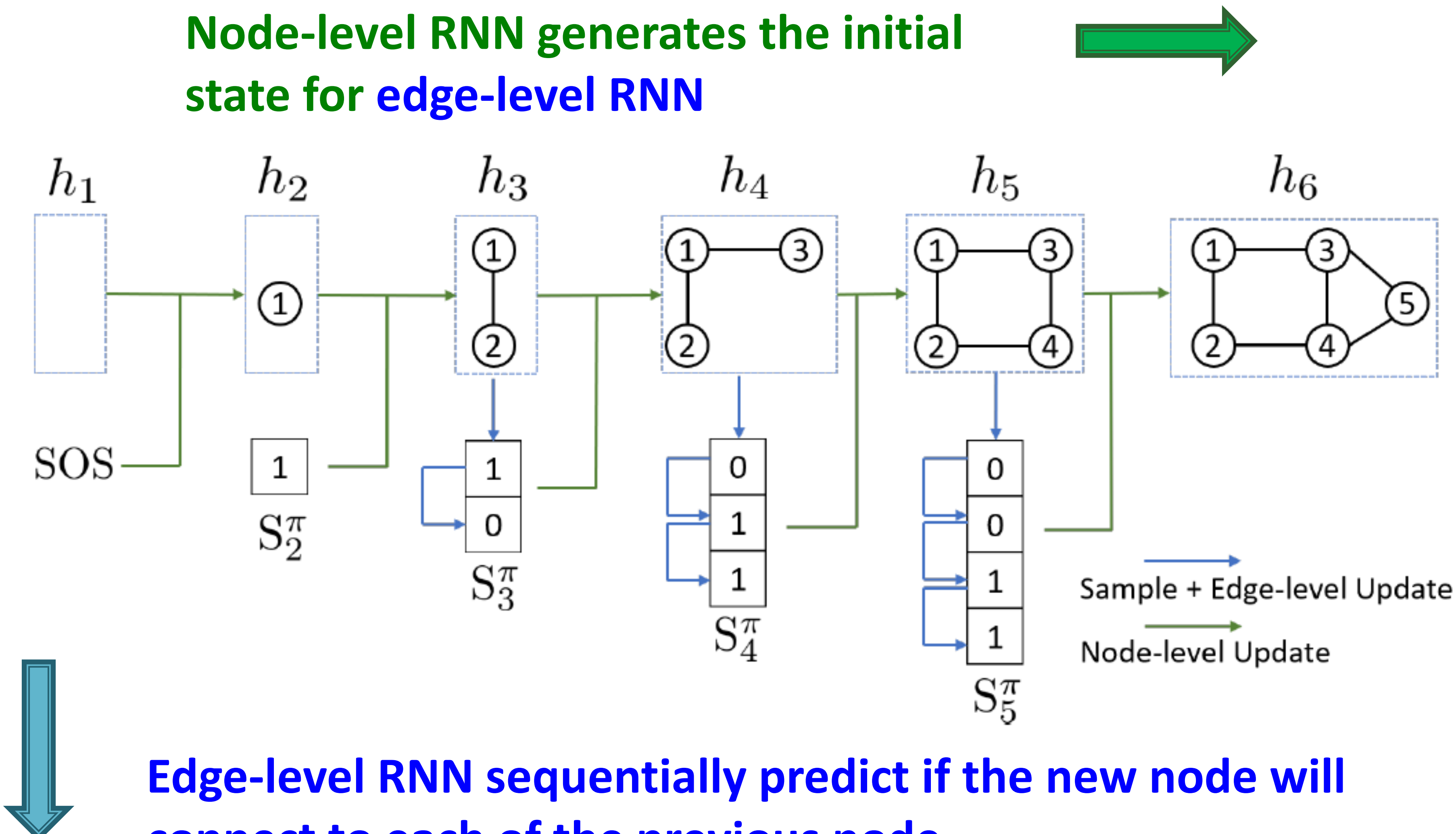
# GraphRNN: Two levels of RNN

- **GraphRNN has a node-level RNN and an edge-level RNN**

- **Relationship between the two RNNs:**
  - Node-level RNN generates the initial state for edge-level RNN
  - Edge-level RNN sequentially predict if the new node will connect to each of the previous node

# GraphRNN: Two levels of RNN

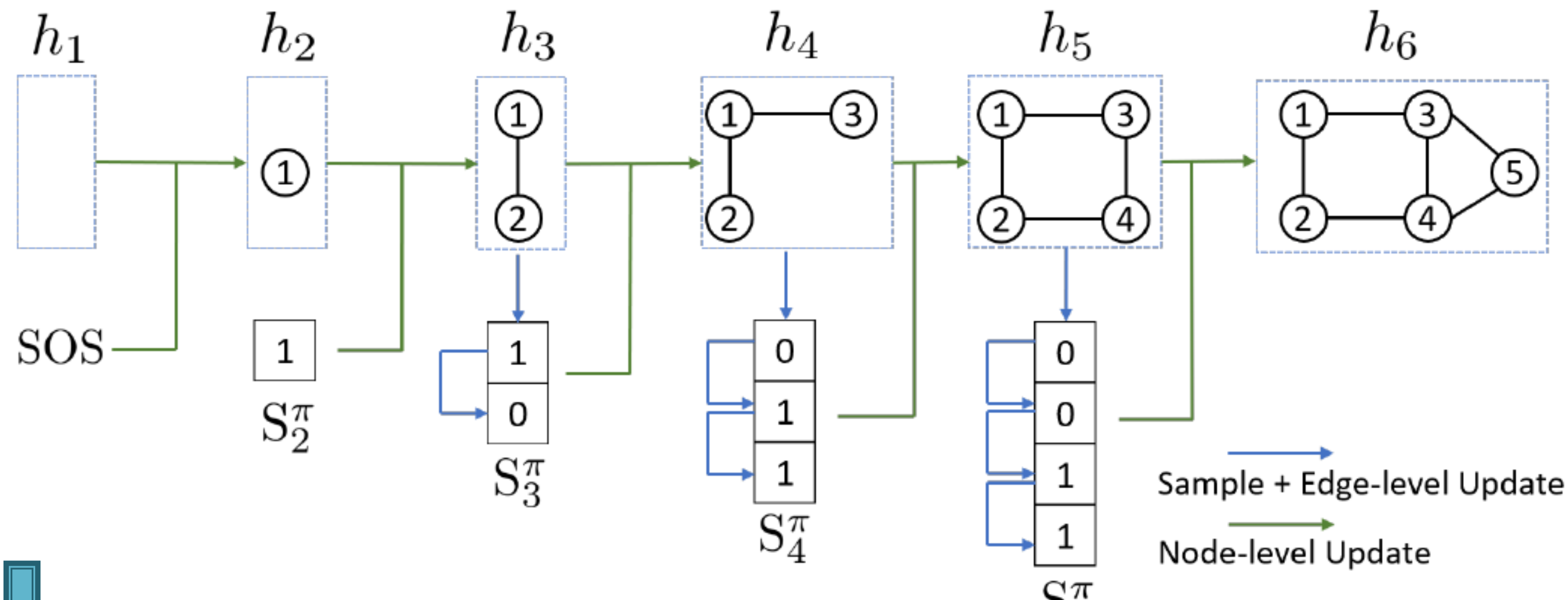
**Node-level RNN generates the initial state for edge-level RNN**




**Edge-level RNN sequentially predict if the new node will connect to each of the previous node**

# GraphRNN: Two levels of RNN

**Node-level RNN generates the initial state for edge-level RNN**




**Next:** How to generate a sequence with RNN?

results

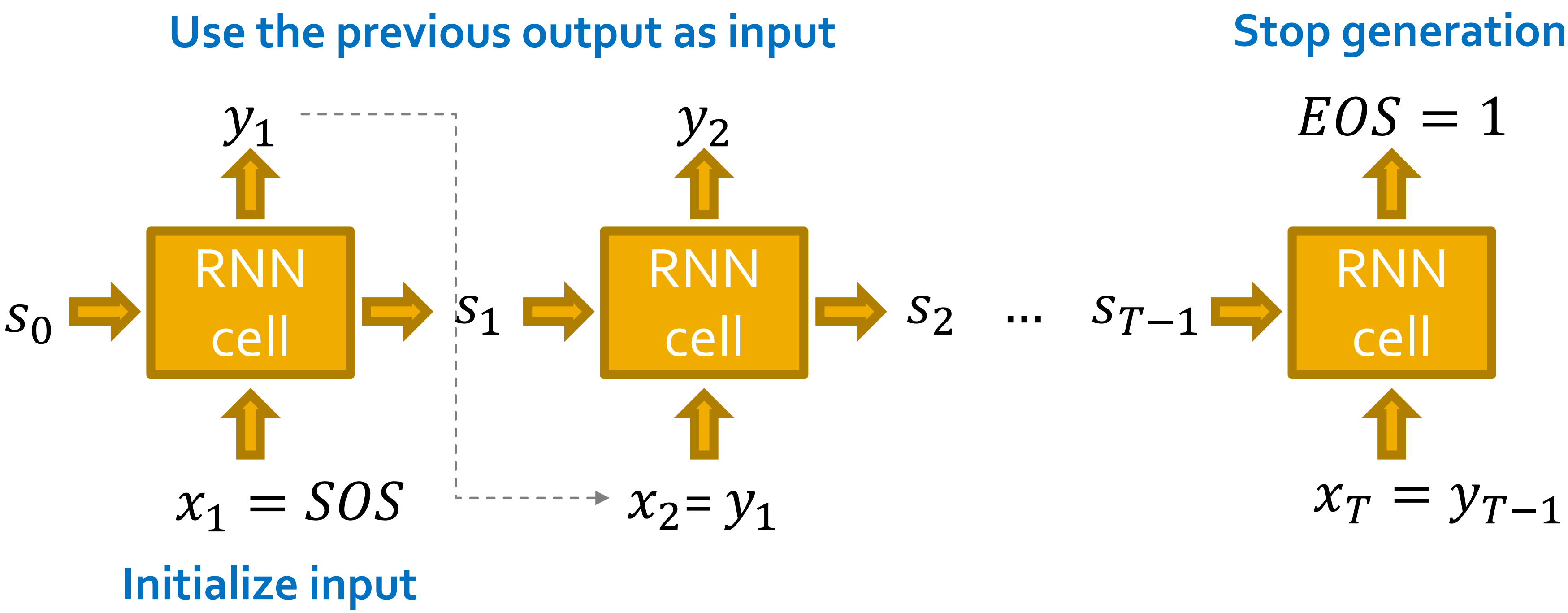
# RNN for Sequence Generation

- **Q:** How to use RNN to generate sequences?
- **A:** Let $x_{t+1} = y_t$ (Use the previous output as input)

- **Q:** How to initialize the input sequence?
- **A:** Use **start of sequence token (SOS)** as the initial input
  - SOS is usually a vector with all zero/ones

- **Q:** When to stop generation?
- **A:** Use **end of sequence token (EOS)** as an **extra** RNN output
  - If output EOS=0, RNN will continue generation
  - If output EOS=1, RNN will stop generation

# RNN for Sequence Generation

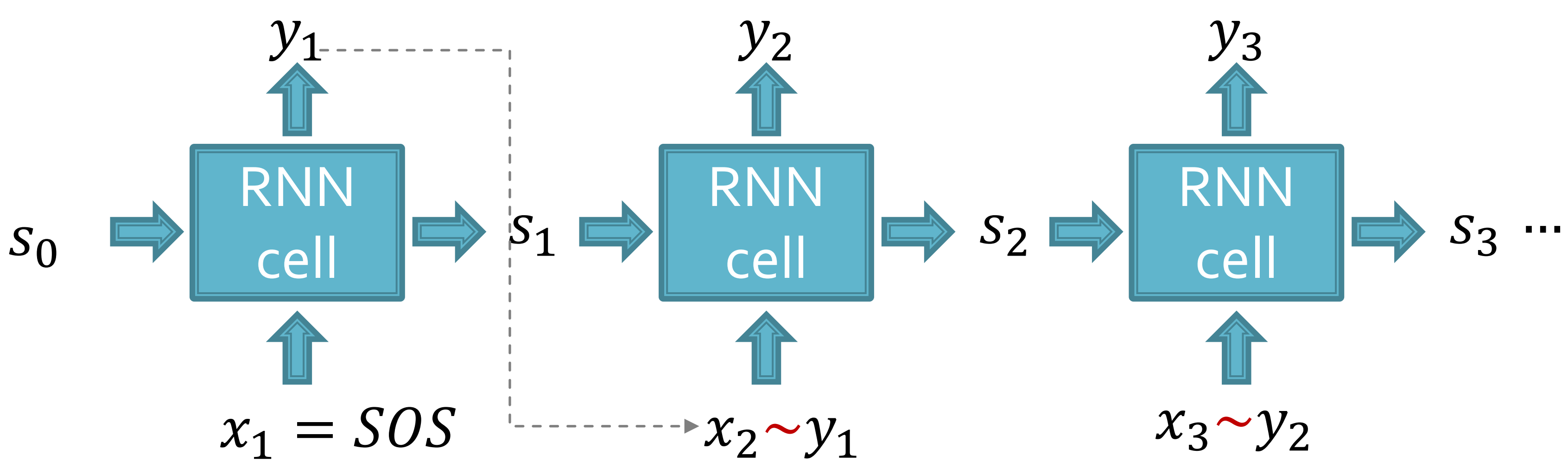



- This is good, but this model is **deterministic**

# Towards Edge-Level RNN

**Consider the Edge-level RNN for now.**

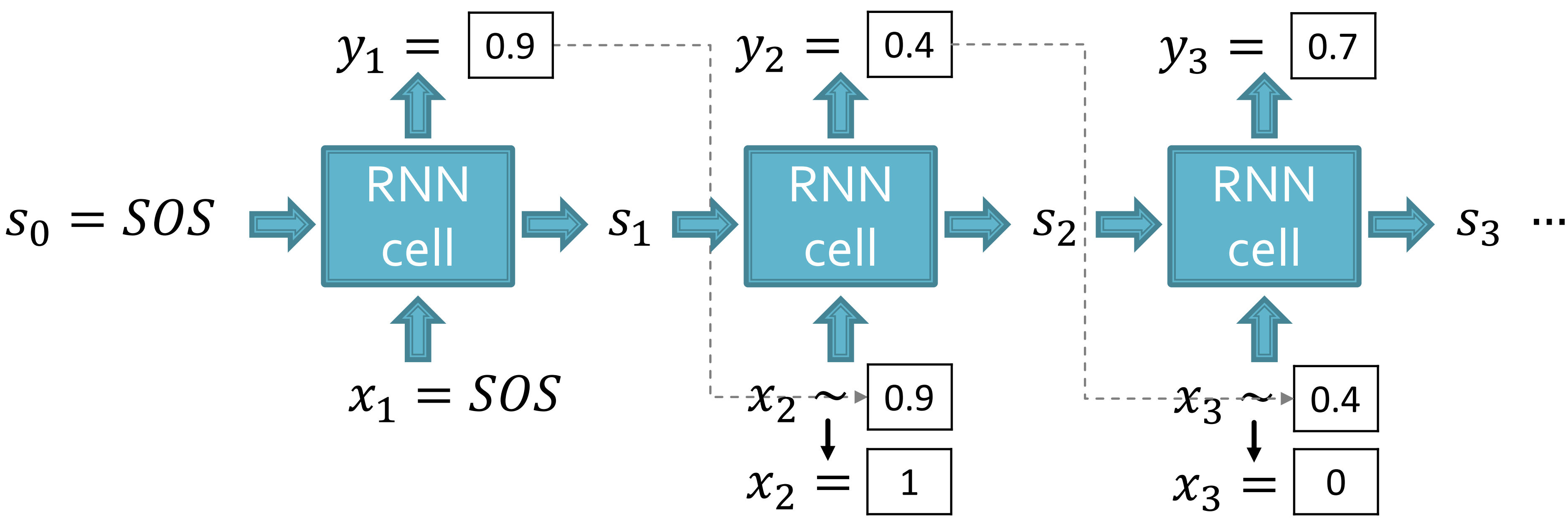
- **Our goal**: Model $\prod_{k=1}^{n} p_{model}(x_t|x_1, \dots, x_{t-1}; \theta)$
- Let $y_t = p_{model}(x_t|x_1, \dots, x_{t-1}; \theta)$
- Then we need to sample $x_{t+1}$ from $y_t$: $x_{t+1} \sim y_t$
  - Each step of RNN outputs a **probability of a single edge**
  - We then sample from the distribution, and feed sample to next step:



# Towards Edge-Level RNN

**Suppose we already have trained the edge-level RNN**

- $y_t$ is a **scalar**, following a **Bernoulli distribution**
- $\boxed{p}$ means value 1 has prob. $p$, value 0 has prob. $1 - p$

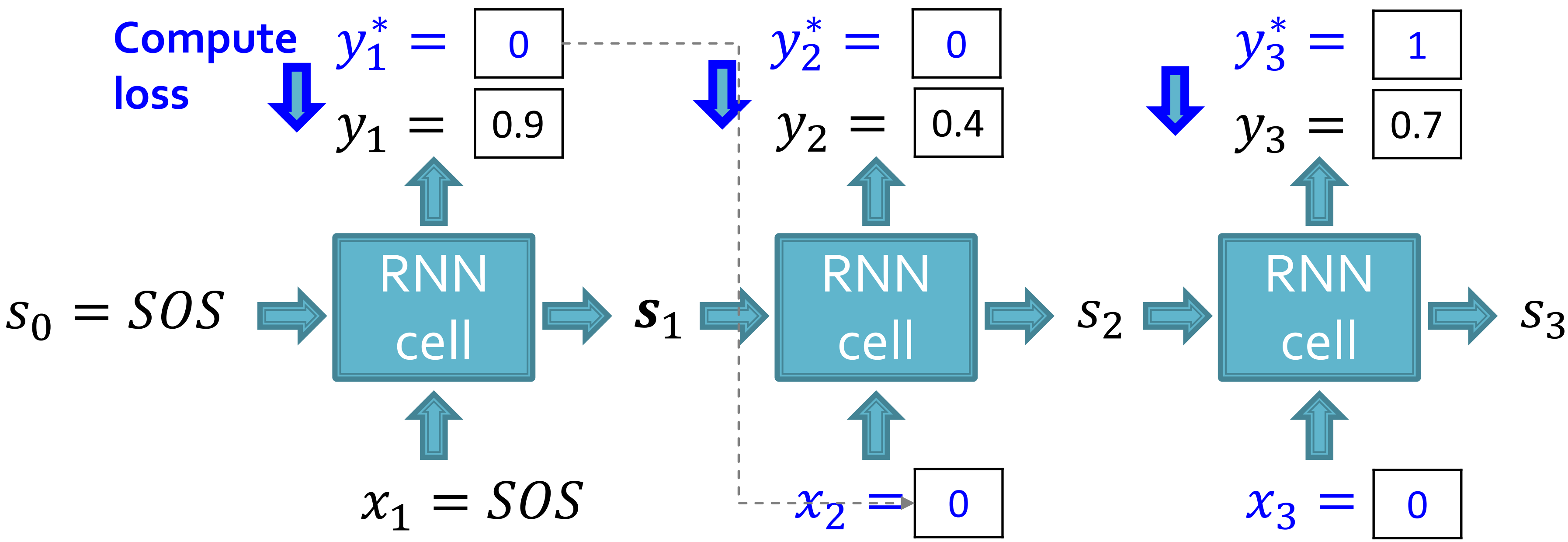



- **How do we use training data $x_1, x_2, \dots, x_n$?**

# Edge-Level RNN at Training Time

**Training the model:**

- We observe a sequence $y^*$ of edges [0,0,1,…]
- **Principle**: **Teacher Forcing** -- Replace input and output by the real sequence



# Edge-Level RNN at Training Time

- **Loss** $L$ **: Binary cross entropy**
- **Minimize:**

$$L = -[y_1^* \log(y_1) + (1 - y_1^*)\log(1 - y_1)]$$

**Compute loss** $\Downarrow$

$y_1^* = \boxed{0}$

$y_1 = \boxed{0.9}$

- If $y_1^* = 1$, we minimize $-\log(y_1)$, making $y_1$ higher
- If $y_1^* = 0$, we minimize $-\log(1 - y_1)$, making $y_1$ lower
- This way, $y_1$ is **fitting** the data samples $y_1^*$
- **Reminder:** $y_1$ is computed by RNN, this loss will **adjust RNN parameters accordingly**, using back propagation!

# Putting Things Together

**Our Plan:**

**(1) Add a new node:** We run Node RNN for a step, and use it output to initialize Edge RNN

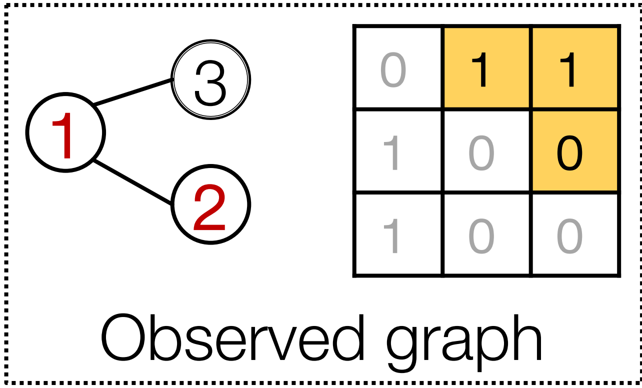
**(2) Add new edges for the new node:** We run Edge RNN to predict if the new node will connect to each of the previous node

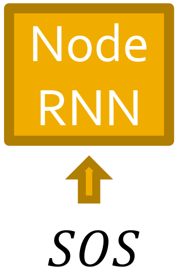
**(3) Add another new node:** We use the last hidden state of Edge RNN to run Node RNN for another step

**(4) Stop graph generation:** If Edge RNN outputs EOS at step 1, we know no edges are connected to the new node. We stop the graph generation.

# Put Things Together: Training

Assuming **Node 1** is in the graph
Now adding **Node 2**

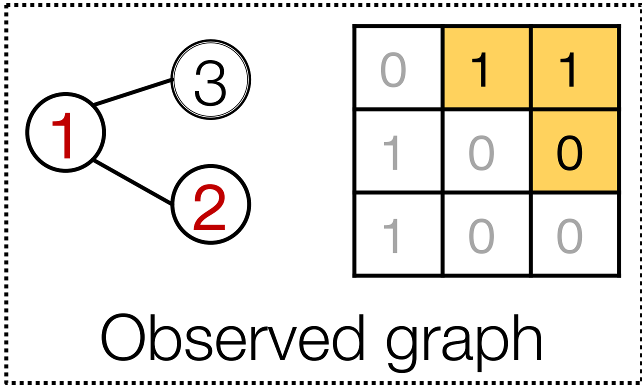


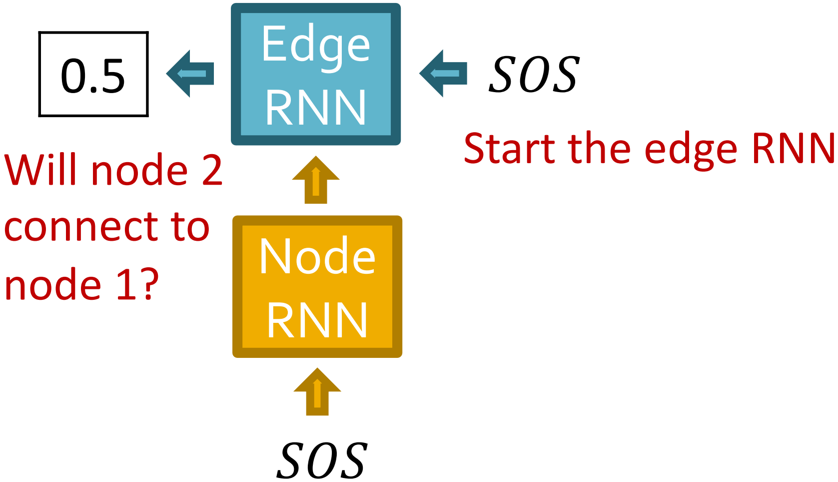




Start the node RNN

# Put Things Together: Training

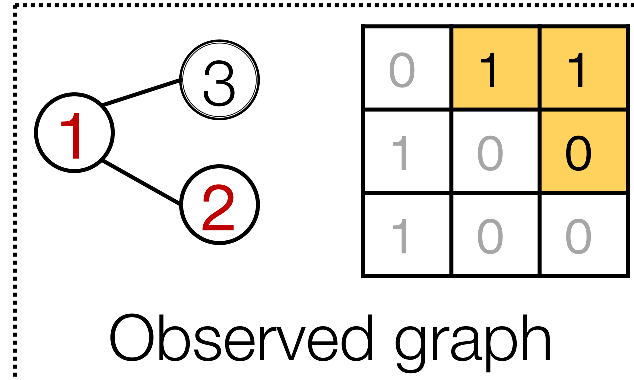
Edge RNN predicts how
**Node 2** connects to **Node 1**

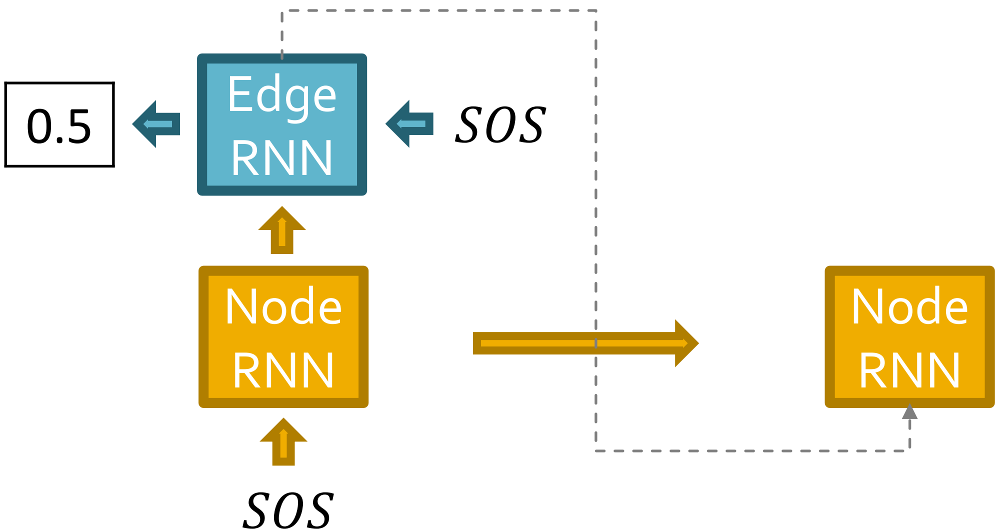


# Put Things Together: Training

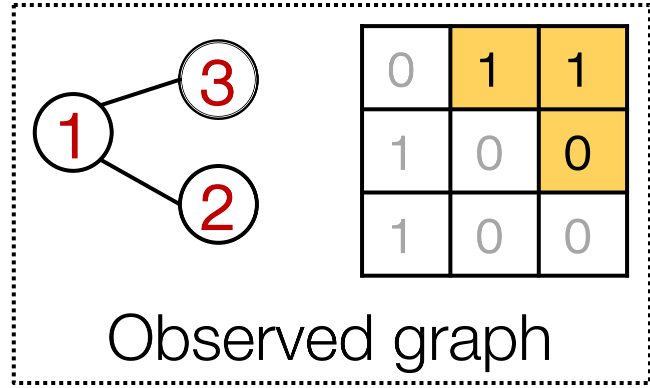
**Update Node RNN** using
**Edge RNN's hidden state**

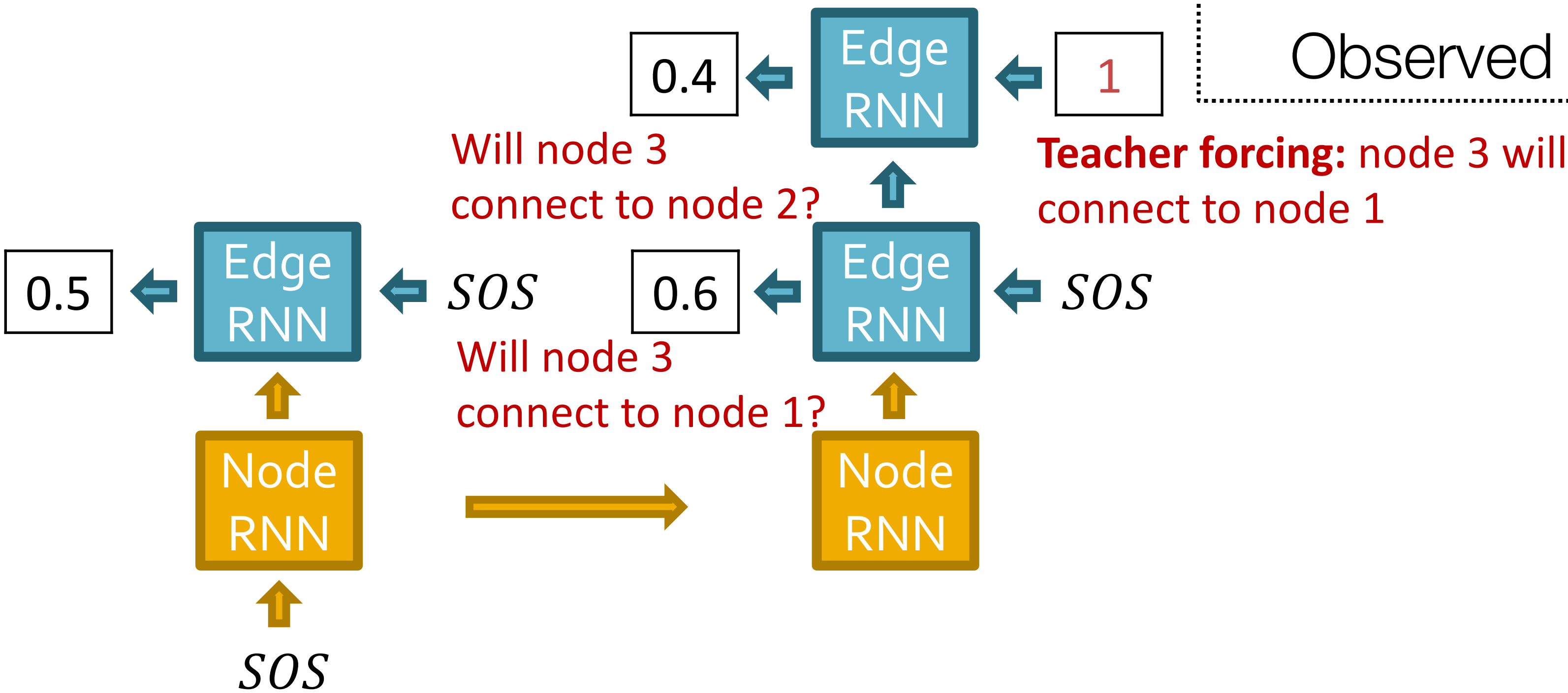


# Put Things Together: Training

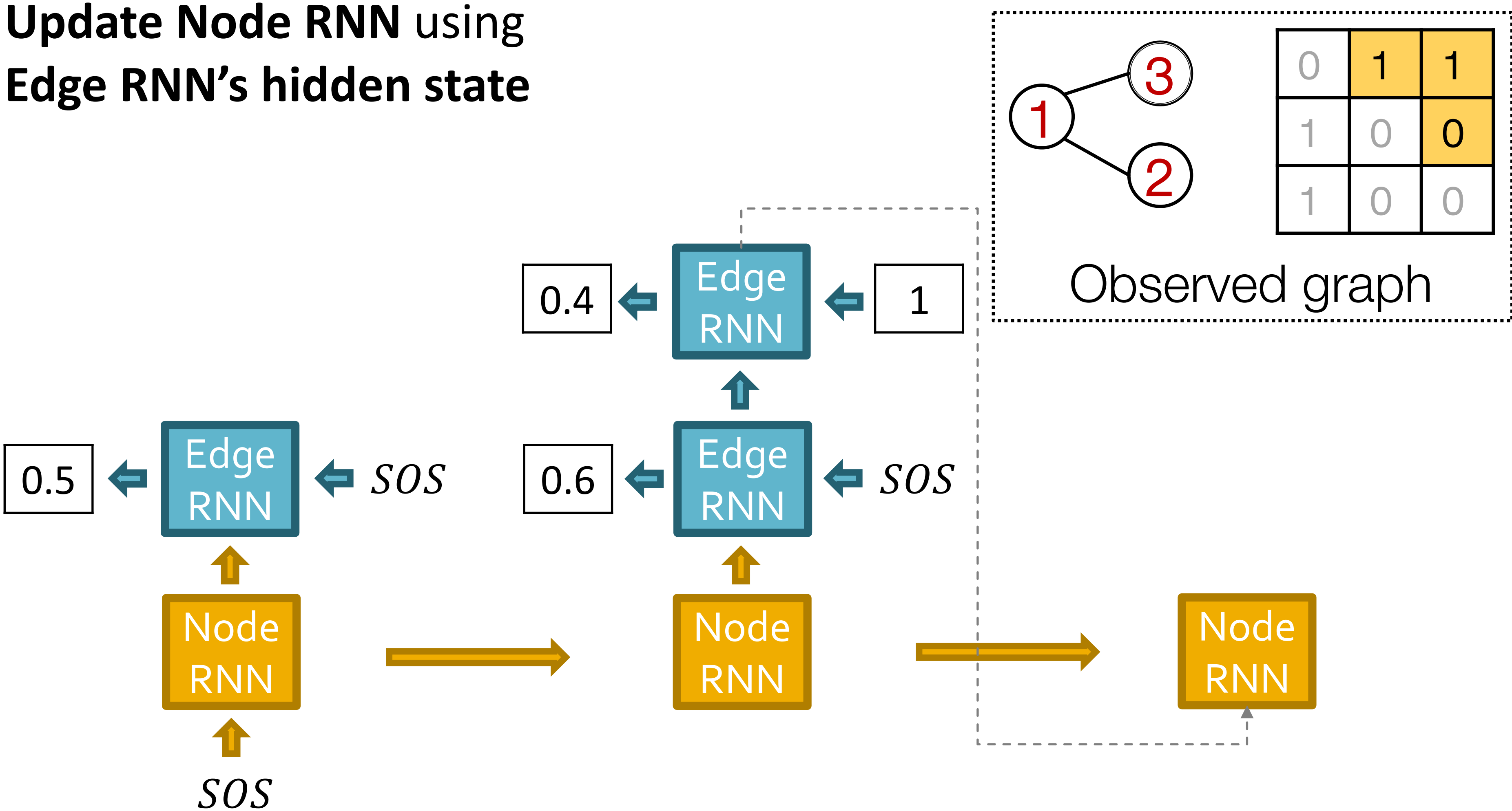
Edge RNN predicts how **Node 3** tries to connects to **Nodes 1, 2**



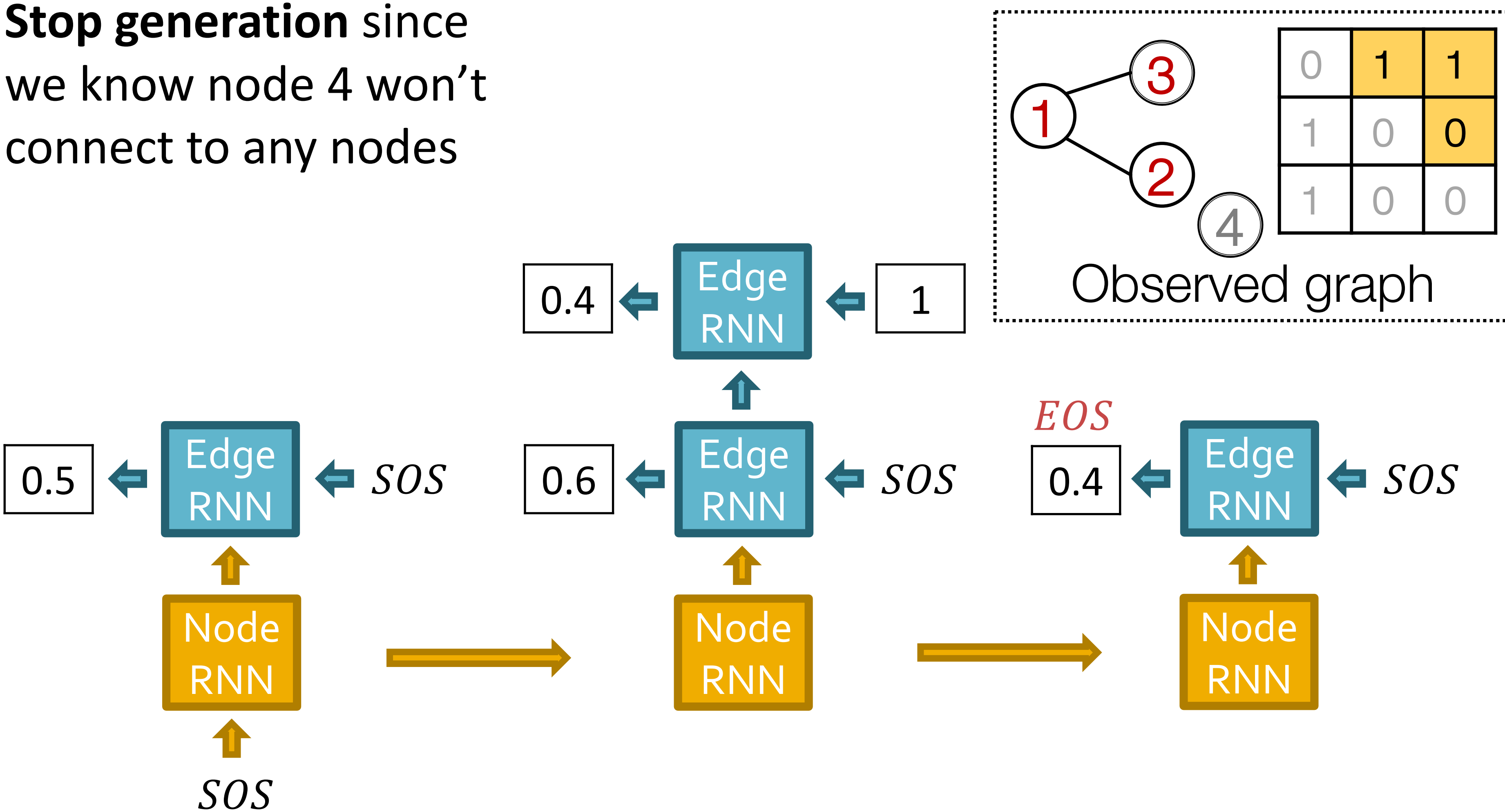
# Put Things Together: Training

**Update Node RNN** using
**Edge RNN's hidden state**



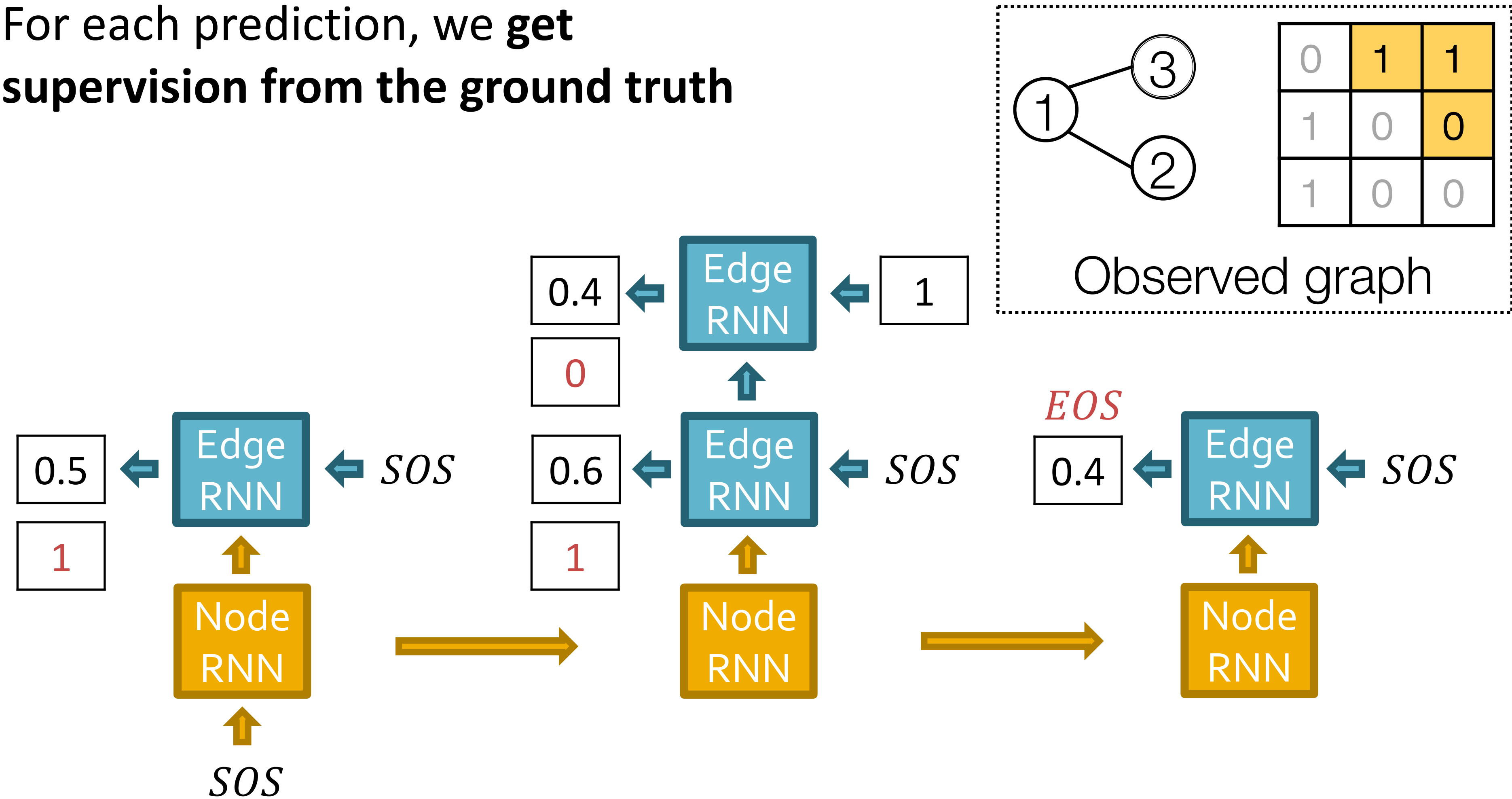
# Put Things Together: Training

**Stop generation** since we know node 4 won't connect to any nodes



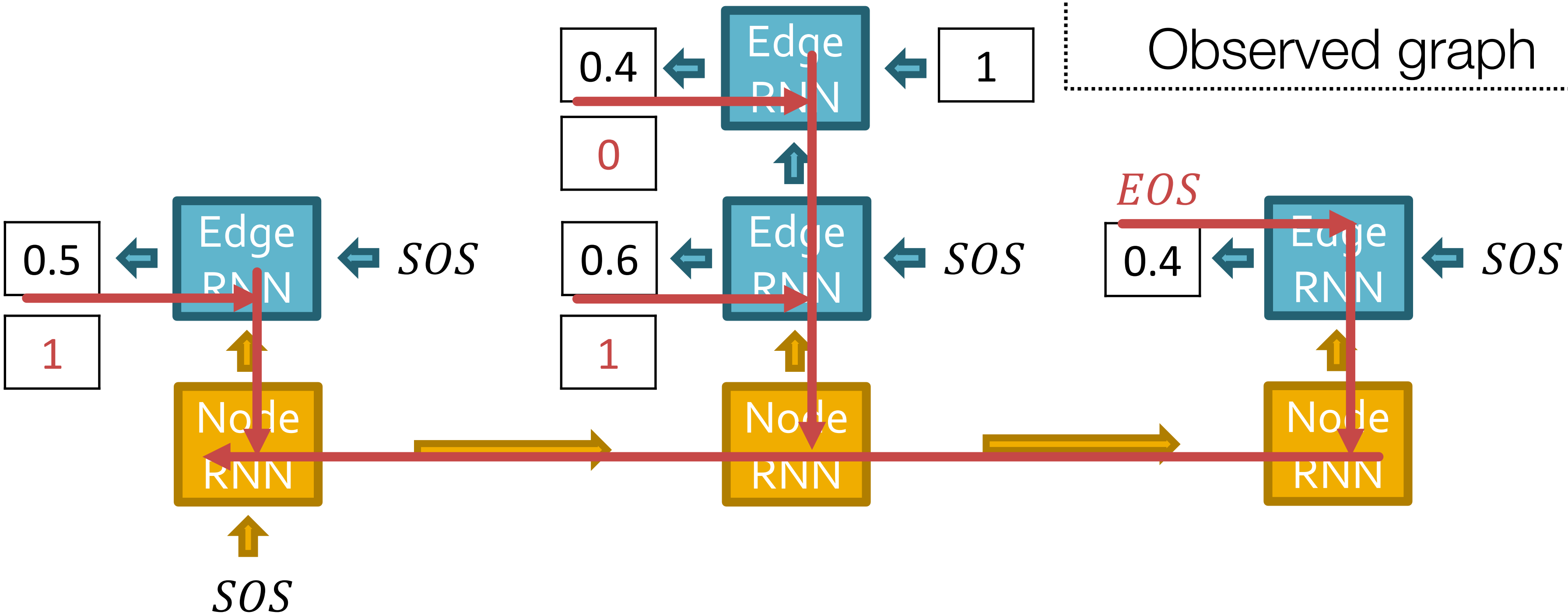
# Put Things Together: Training

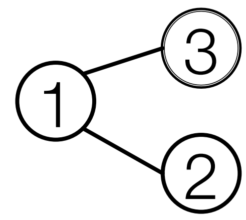
For each prediction, we **get supervision from the ground truth**



# Put Things Together: Training

**Backprop through time:**
Gradients are **accumulated across time steps**

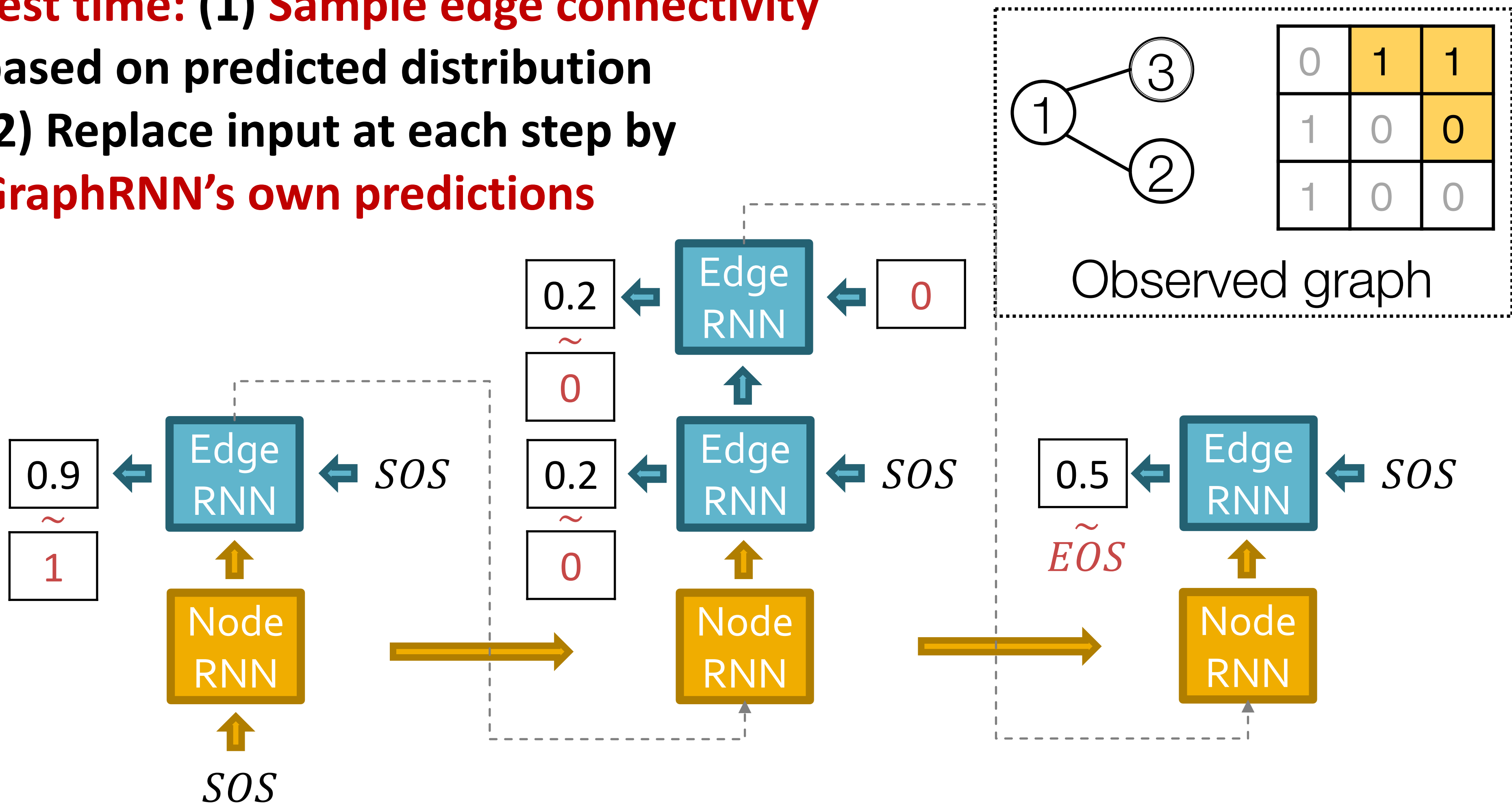



| | | |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 0 |

# Put Things Together: Test

**Test time: (1) Sample edge connectivity based on predicted distribution (2) Replace input at each step by GraphRNN's own predictions**

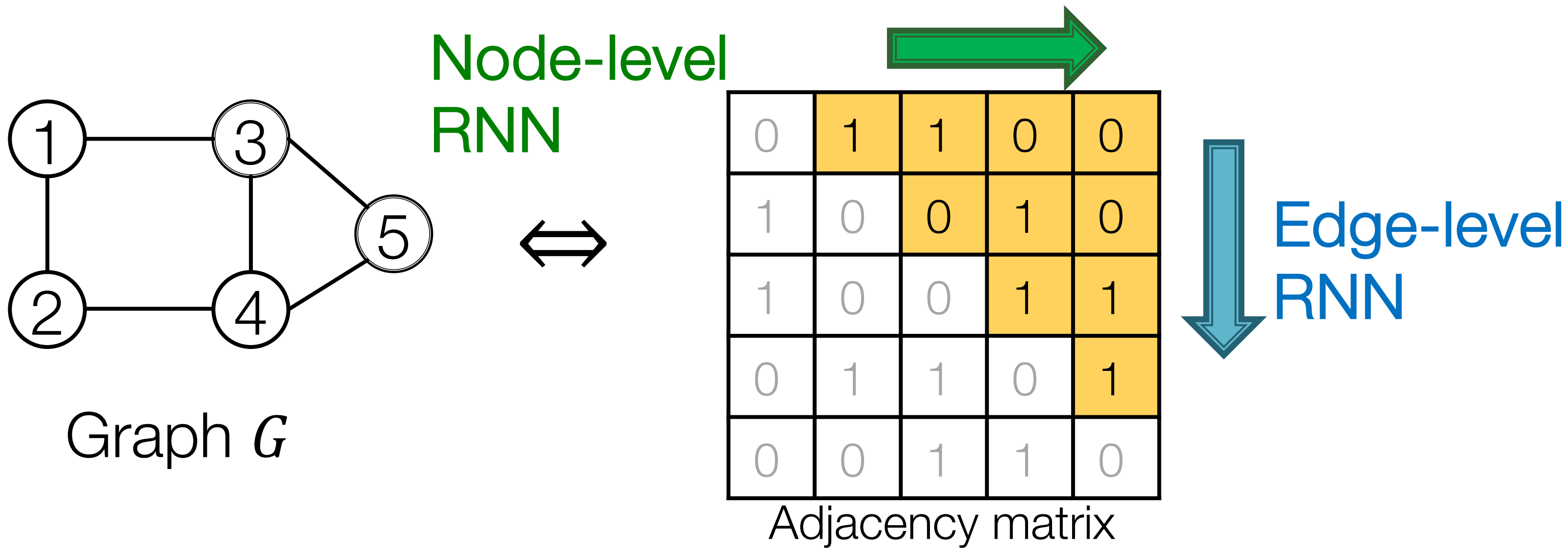


# GraphRNN: Two levels of RNN

**Quick Summary of GraphRNN:**

- Generate a graph by generating a two-level sequence
- Use RNN to generate the sequences

- **Next**: Making GraphRNN tractable, proper evaluation

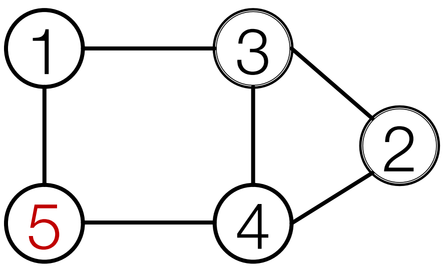


# Stanford CS224W: Scaling Up and Evaluating Graph Generation

CS224W: Machine Learning with Graphs
Jure Leskovec, Stanford University
http://cs224w.stanford.edu

# Issue: Tractability

- Any node can connect to any prior node
- Too many steps for edge generation
  - Need to generate full adjacency matrix
  - Complex too-long edge dependencies




Random node ordering:
Node 5 may connect to any/all previous nodes

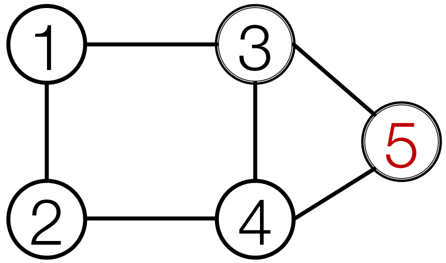
**"Recipe" to generate the left graph:**

- Add node 1
- Add node 2
- Add node 3
- Connect 3 with 2 and 1
- Add node 4
- …

How do we limit this complexity?

# Solution: Tractability via BFS

- **Breadth-First Search node ordering**



BFS ordering

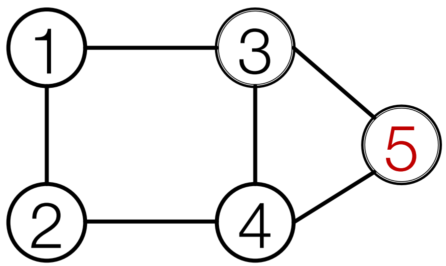
**"Recipe" to generate the left graph:**

- Add node 1
- Add node 2
- Connect 2 with 1
- Add node 3
- Connect 3 with 1
- Add node 4
- Connect 4 with 3 and 2

- **BFS node ordering:**
  - Since Node 4 doesn't connect to Node 1
  - We know all Node 1's neighbors have already been traversed
  - Therefore, Node 5 and the following nodes will never connect to node 1
  - We only need memory of 2 "steps" rather than $n - 1$ steps

# Solution: Tractability via BFS

- **Breadth-First Search node ordering**



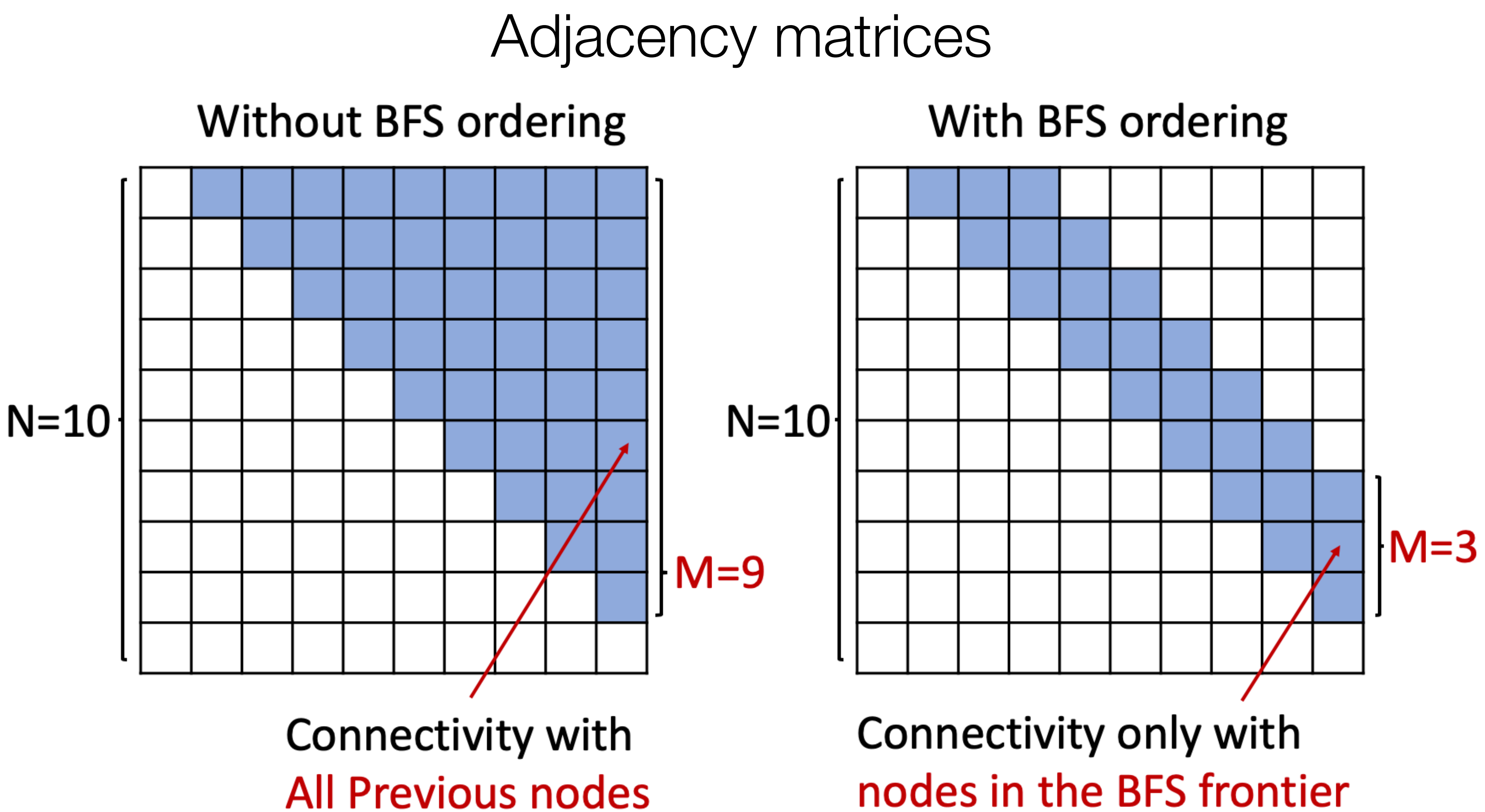
BFS ordering

BFS node ordering: Node 5 will never connect to node 1 (only need memory of 2 "steps" rather than $n - 1$ steps)

- **Benefits:**
  - Reduce possible node orderings
    - From $O(n!)$ to number of distinct BFS orderings
  - Reduce steps for edge generation
    - Reducing number of previous nodes to look at

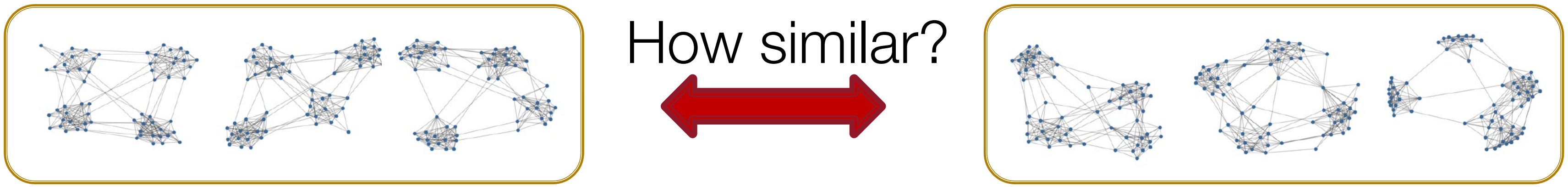
# Solution: Tractability via BFS

- **BFS reduces the number of steps for edge generation**



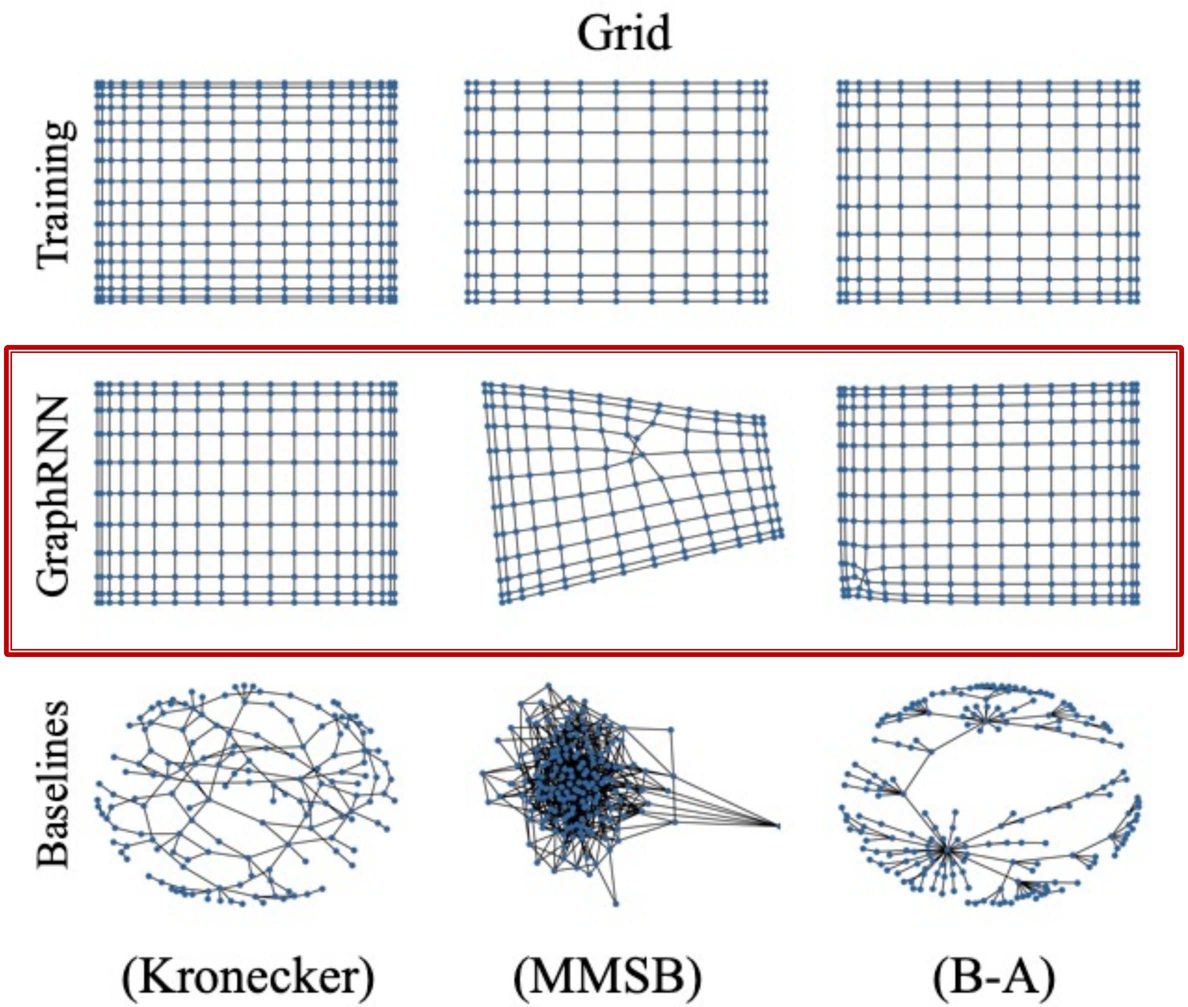
# Evaluating Generated Graphs

- **Task: Compare two sets of graphs**

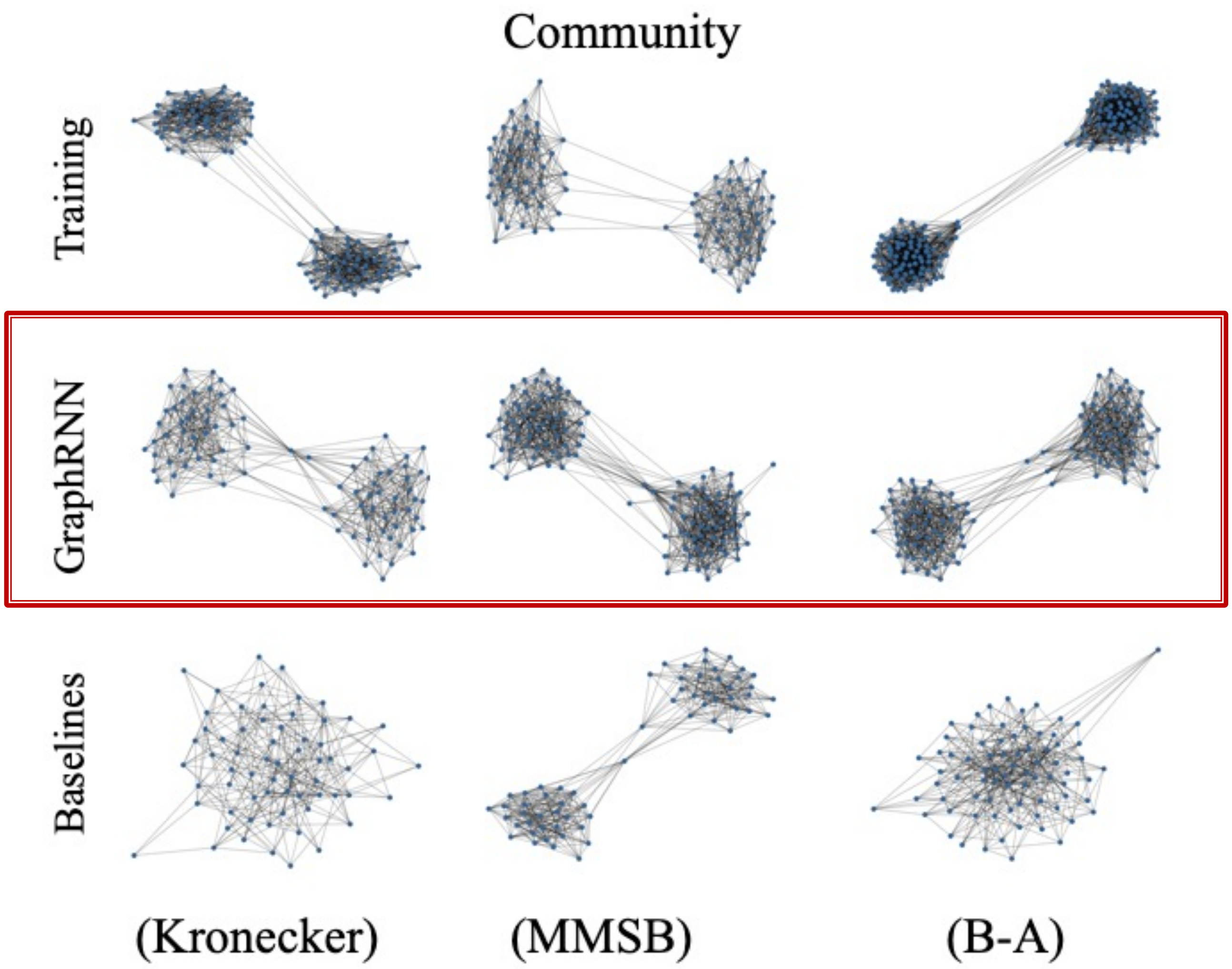



- **Goal:** Define similarity metrics for graphs
- **Solution**
  - **(1) Visual similarity**
  - **(2) Graph statistics similarity**

# (1) Visual Similarity



# (1) Visual Similarity

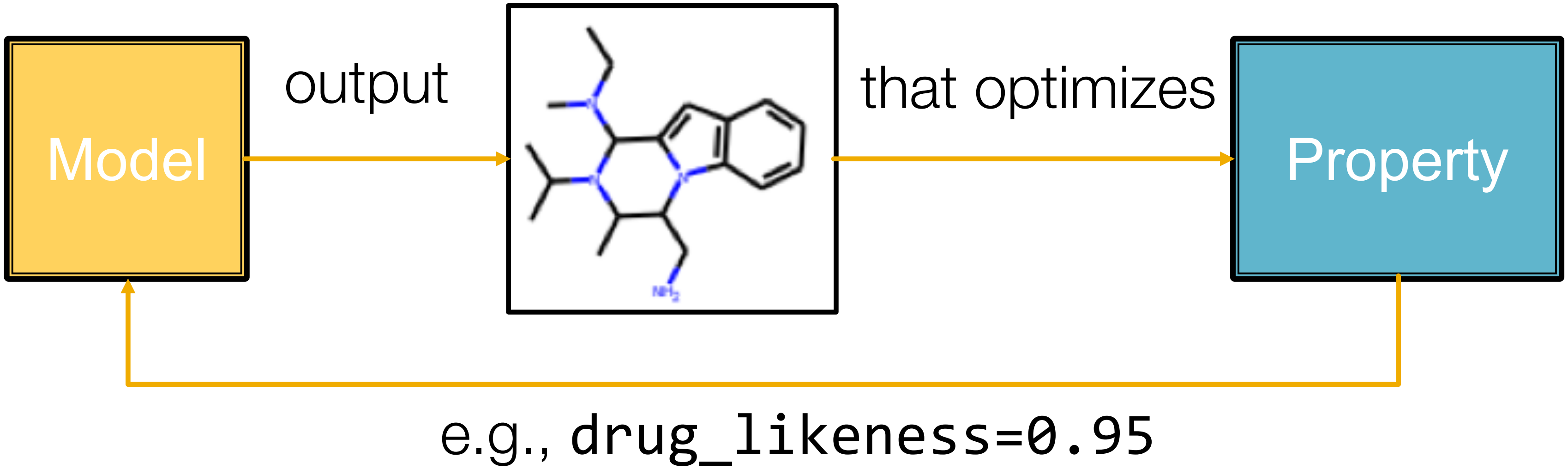


# Stanford CS224W: Application of Deep Graph Generative Models to Molecule Generation

CS224W: Machine Learning with Graphs
Jure Leskovec, Stanford University
http://cs224w.stanford.edu

# Application: Drug Discovery

**Question:** Can we learn a model that can generate **valid** and **realistic** molecules with **optimized** property scores?




[Graph Convolutional Policy Network for Goal-Directed Molecular Graph Generation](). J. You, B. Liu, R. Ying, V. Pande, J. Leskovec. *Neural Information Processing Systems (NeurIPS)*, 2018.

# Goal-Directed Graph Generation

**Generating graphs that:**

- **Optimize a given objective** (High scores)
  - e.g., drug-likeness
- **Obey underlying rules** (Valid)
  - e.g., chemical validity rules
- **Are learned from examples** (Realistic)
  - Imitating a molecule graph dataset
    - We have just covered this part

# The Hard Part:

## Generating graphs that:

- **Optimize a given objective** (High scores)

**Including a "Black-box" to Graph Generation:**

**Objectives like drug-likeness are governed by physical law which is assumed to be unknown to us.**

- Covered this part when introducing GraphRNN

# Idea: Reinforcement Learning

- A ML agent **observes** the environment, takes an **action** to interact with the environment, and receives positive or negative **reward**
- The agent then **learns from this loop**
- **Key idea**: Agent can directly learn from environment, which is a **blackbox** to the agent

# Solution: GCPN

**Graph Convolutional Policy Network (GCPN)** combines graph representation + RL

**Key component of GCPN:**

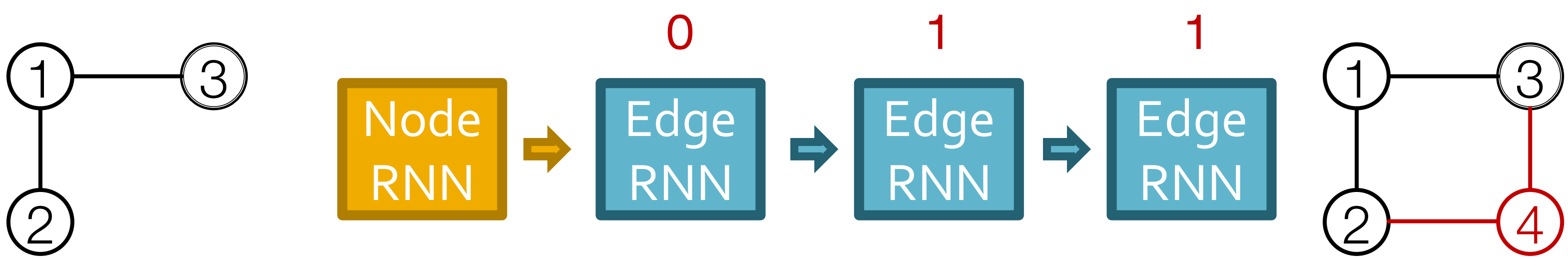
- Graph Neural Network captures graph structural information
- Reinforcement learning guides the generation towards the desired objectives
- Supervised training imitates examples in given datasets

# GCPN vs. GraphRNN

- **Commonality of GCPN & GraphRNN:**
  - Generate graphs sequentially
  - Imitate a given graph dataset
- **Main Differences:**
  - GCPN uses **GNN** to predict the generation action
    - **Pros:** GNN is more expressive than RNN
    - **Cons:** GNN takes longer time to compute than RNN
  - GCPN further uses **RL** to direct graph generation to our goals
    - RL enables goal-directed graph generation

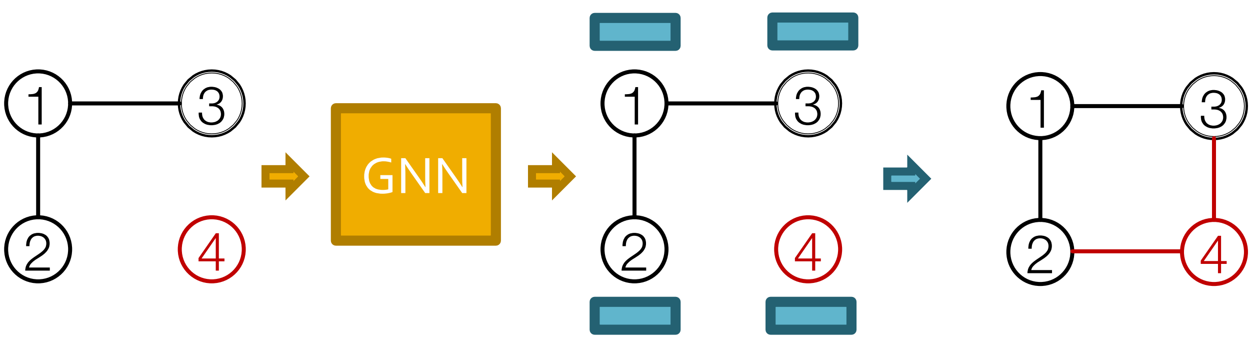
# GCPN vs. GraphRNN

- **Sequential graph generation**
- **GraphRNN**: predict action based on **RNN hidden states**




RNN hidden state captures the generated graph so far

- **GCPN:** predict action based on **GNN node embeddings**



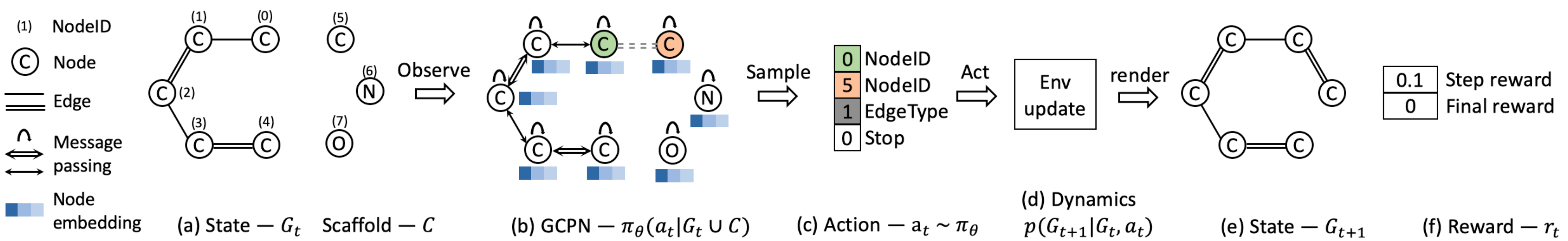

**Recall the link prediction head:**

$\text{Head}_{edge}(\mathbf{h}_u^{(L)}, \mathbf{h}_v^{(L)}) := \text{Linear}(\text{Concat}(\mathbf{h}_u^{(L)}, \mathbf{h}_v^{(L)}))$

Node embeddings

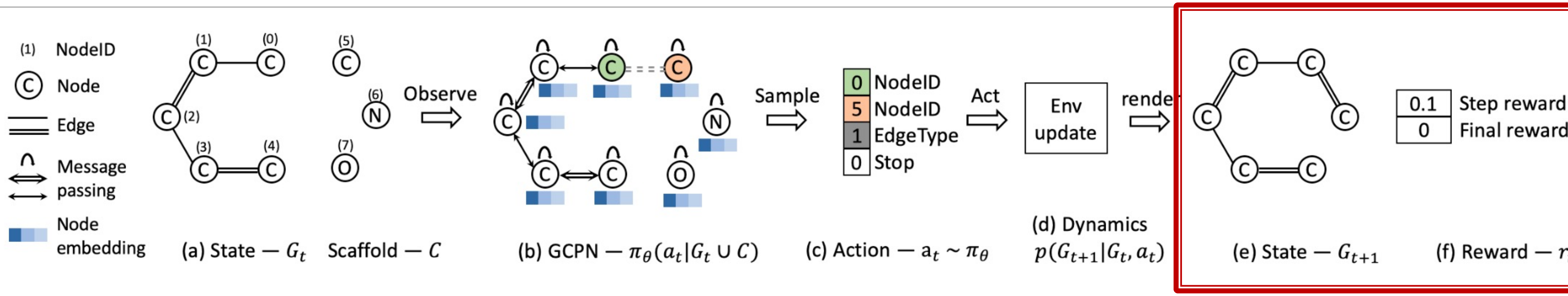
Predict potential links using node embeddings

# Overview of GCPN




- **(a)** Insert nodes
- **(b,c)** Use GNN to predict which nodes to connect
- **(d)** Take an action (check chemical validity)
- **(e, f)** Compute reward

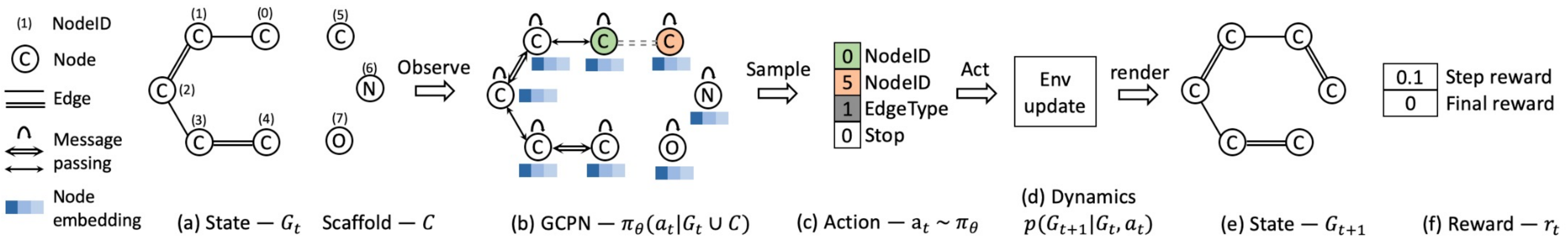
# How Do We Set the Reward?




- **Step reward:** Learn to take valid action
  - At each step, assign small positive reward for valid action
- **Final reward:** Optimize desired properties
  - At the end, assign positive reward for high desired property

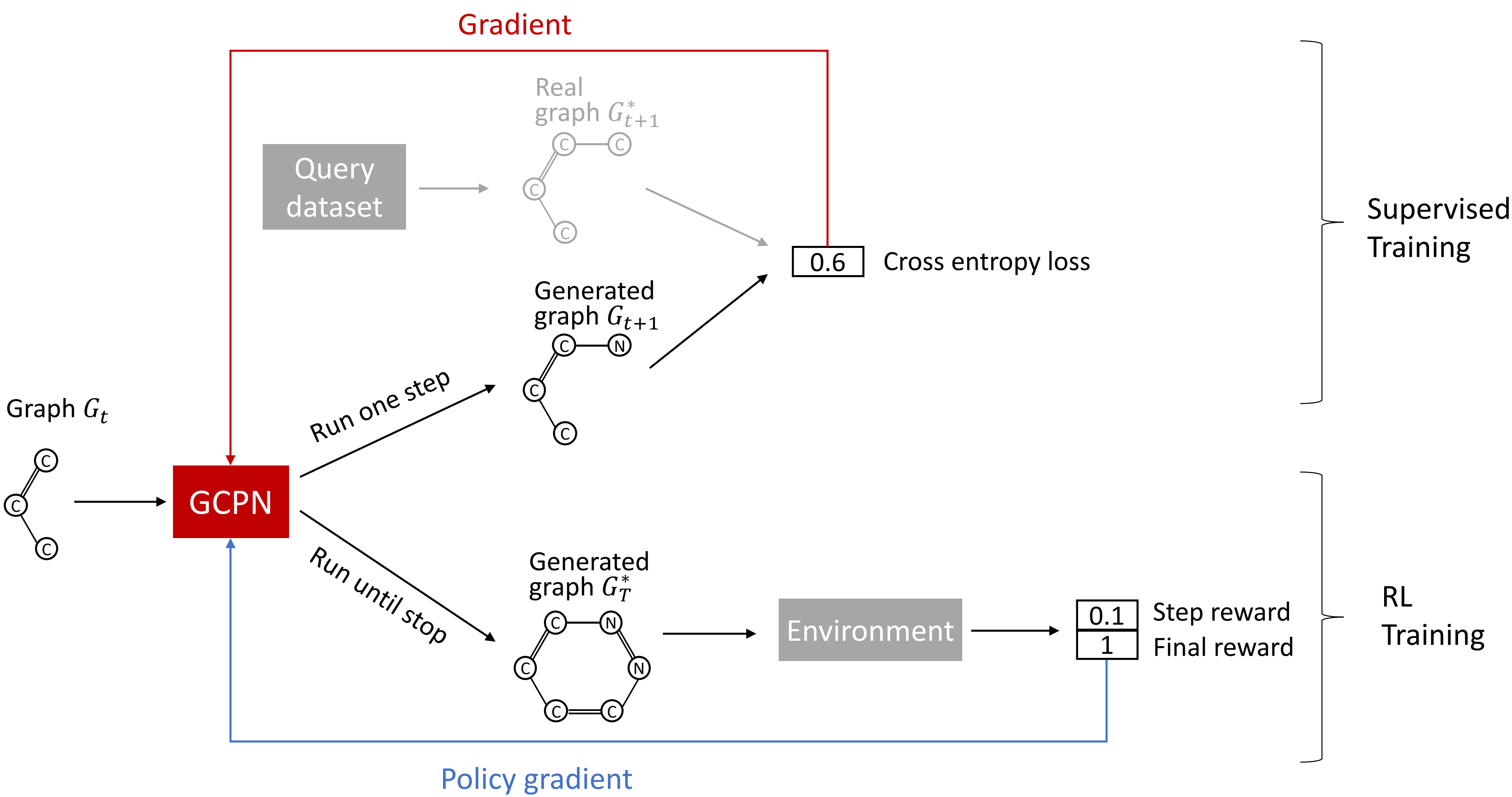
**Reward = Final reward + Step reward**

# How Do We Train?




- **Two parts:**
- **(1) Supervised training:** Train policy by imitating the action given by real observed graphs. Use gradient.
  - We have covered this idea in GraphRNN
- **(2) RL training:** Train policy to optimize rewards. Use standard policy gradient algorithm.
  - Refer to any RL course, e.g., CS234

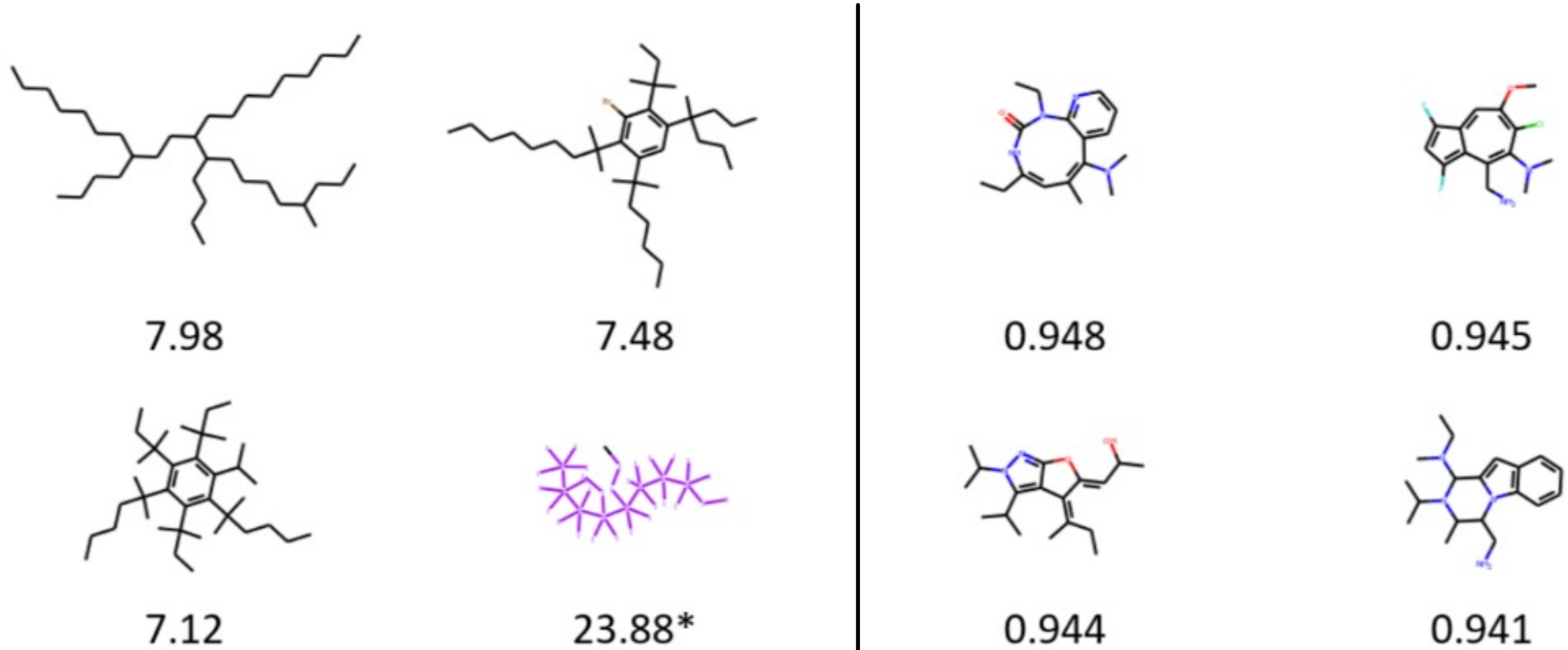
# Training GCPN



# Qualitative Results

**Visualization of GCPN graphs:**

- Property optimization Generate molecules with high specified property score




(a) Penalized logP optimization

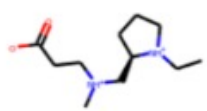
(b) QED optimization

# Qualitative Results

## Visualization of GCPN graphs:

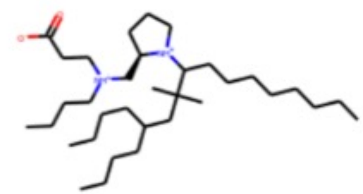
- Constrained optimization: Edit a given molecule for a few steps to achieve higher property score

Starting structure

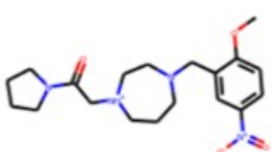


-8.32

Finished structure

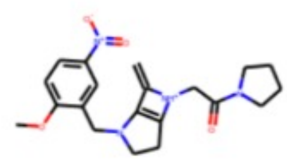


-0.71



-5.55

**Increase the solubility in octanol**



-1.78

(c) Constrained optimization of penalized logP

# Summary of Graph Generation

- Complex graphs can be successfully generated via sequential generation using deep learning
- Each step a decision is made based on hidden state, which can be
  - Implicit: vector representation, decode with RNN
  - Explicit: intermediate generated graphs, decode with GCN
- Possible tasks:
  - Imitating a set of given graphs
  - Optimizing graphs towards given goals